DISTRIBUTION OF THE INTERSTELLAR GRAINS

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Abstract

For 40 cepheids in $l=60\sim215°$, there exists fairly strong correlation between colour excesses and quantities of neutral hydrogen atoms, suggesting that the interstellar grains and the neutral hydrogen atoms may coexist. In wider regions on the galactic plane, colour excess per unit distance seems to show a linear decrease outwardly.

If we adopt van de Hulst's model of grain formation, this decrease yields outward decrease of grain density. But both decreases cannot be connected directly. Sizes of grains give negative effect to the decrease of colour excess inside about $R=10$ kpc. The mean grain size becomes larger in the outer parts of the galactic plane owing to lower density which is favourable for grains to grow excessively.

1. Introduction.

It has long been suspected that the interstellar dust particles coexist with gas, but the comparison of the observational results on star counts and on 21 cm emission gives conflicting aspects, showing negative or positive correlations for various directions, e.g. van de Hulst, Muller and Oort(1954), Lilley(1955), Heeschen(1955), Bok(1955), and Davies(1956) who suggests that this incosistency dues to the difference in referred distances for the direction studied. Moreover, most of the 21 cm studies in connection with the interrelations with dust have been carried out on some specified regions on the celestial sphere.

Cepheids are favourable to investigate the distribution of the interstellar grains since they scatter widely on the galactic plane, and their distances can be estimated with sufficient accuracy. In the following sections, correlations on colour excess are evaluated, and the general tendency of the colour excess is investigated to estimate sizes, numbers, and densities of the grains.

2. Correlations on Colour Excess.

For galactic longitudes between $l=60°\sim215°$, Westerhout(1957) gives the maps of density contour of the neutral hydrogen atoms perpendicular to the galactic plane for every two and a half or five degrees in longitudes. In this sector, there are 12 and 32 cepheids whose positions have been evaluated by Gascoign and Eggen(1957), and by Walraven, Muller and Oosterhoff(1958), respectively.

At two or four longitudes on both sides of each cepheid, numbers of neutral hydrogen atoms between the sun and the corresponding point of latitude b and distance r are counted on Westerhout's chart, from which the actual number of the hydrogen atoms on the line of sight to the cepheid are obtained by interpolation. The values of colour excesses $(P-V)_E$ by Eggen, Gascoign and Burr(1957) are reduced to the *SCI*–system in which the measurement of colour excesses were made by Walraven, Muller and Oosterhoff.

We denote by E the colour excess, and by N_H the number of neutral hydrogen which lie between us and the star. Put

$$\tau_H = N_H /(3.08 \times 10^{21}), \qquad (1)$$

which is plotted against E in Fig. 1. The correlation coefficient is obtained:

$$r_{\tau E} = 0.68 \pm 0.10.$$

In this computation, weights are assigned to be 0.5 for the stars whose values of E_{SCI} are remarked as (:) in the original data, and for the two stars which are common these two observations, taking weight 1 for the remainders.

This fairly large value of coefficient suggests the coexistence of the interstellar dust and gas. However, it may be attributed to the fact that they are both functions of the distances from us, respectively. Or, the larger the distance the more the number of grains, and the more the numbers of the hydrogen atoms, too. In order to investigate this effect, following two correlation coefficients are evaluated:

$$r_{Er} = 0.47 \pm 0.15,$$

$$r_{\tau r} = 0.68 \pm 0.11.$$

Clearly, E and τ_H do not increase taking the distance r as a parameter. Though the both colour observations here employed are elaborate, they cannot be free from errors more or less in the estimated distances. And it is less probable to evaluate τ_H larger owing to the error in distance than to evaluate lesser, since the neutral hydrogen atoms occupy the limited regions. Hence, the actual correlation coefficient $r_{\tau E}$ may be fairly larger than the value obtained above.

The regressions of E on τ_H, and τ_H on E are

$$E = 0.29\ \tau_H + 0.15 \qquad \text{with } \sigma_E = 0.16,$$

$$\tau_H = 1.59\ E + 0.23 \qquad \text{with } \sigma_\tau = 0.36,$$

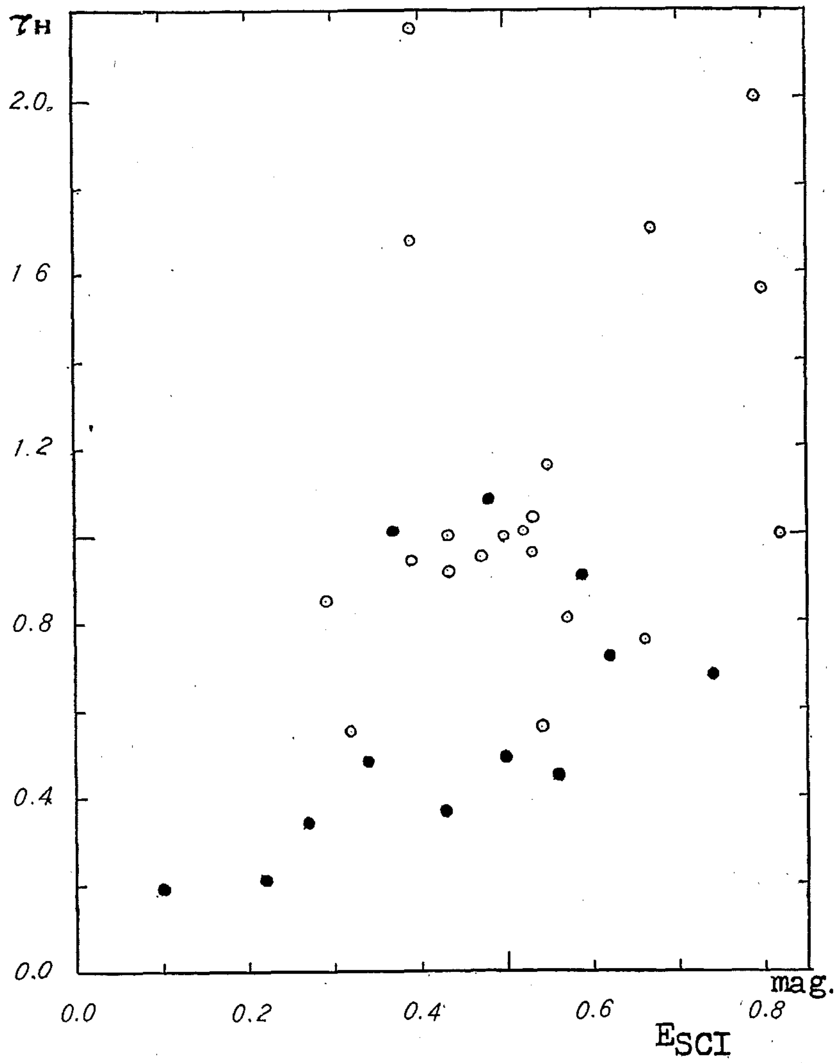

Fig. 1 Relations between Colour Excesses and Numbers of Neutral Hydrogen.
○=Eggen et al. (1957),
●=Walraven et al. (1958).

respectively, whose weighted mean is:

$$\tau_H = 2.16E - 0.01 \qquad \text{with } \sigma = 0.64, \tag{2}$$

Omitting the constant term, we have by (1),

$$E = 1.50 \times 10^{-22} N_H. \tag{3}$$

Denoting by k mass absorption coefficient, by M total mass of the grains on the line of sight to the star, we have

$$E = cM \{k(\lambda_1) - k(\lambda_2)\}, \tag{4}$$

where λ_1, λ_2 are effective wavelengths for the colour, and $c = \log_{10}e^{5/2} = 1.09$. If we write

$$M = \alpha N_H, \tag{5}$$

then, α, the mass of the interstellar grains coexisting with one neutral hydrogen atom, is expressed by

$$\alpha = \frac{M}{N_H} = \frac{1.50 \times 10^{-22}}{1.09} \frac{1}{k(\lambda_1) - k(\lambda_2)}. \tag{6}$$

Assuming $k = k_0 \lambda^{-0.9}$ and $k = 1.4 \times 10^{-5}$ mag cm^2 gr^{-1}, which is given by Greenstein(1938) for the dielectric particle of refractive index $m=4/3$ at 4400A, we obtain

$$\alpha = 0.51 \times 10^{-26} \text{ gr per 1 H-atom.}$$

If we assume the mass of grain to be 10^{-13} gr (Allen, 1955), we have

Number of grains per 1 H–atom$=0.5\times10^{-13}$,

or, Number of grains per unit volume$=0.3\times10^{-13}$.

These values are fairly smaller than the values currently adopted, e.g. 2×10^{-13} grains per unit volume, and shall be treated from the other standpoint in the following sections.

3. Colour Excess as a Function of Galactocentric Distance.

For the sector between longitudes $l=160°\sim40°$, the contour map of the hydrogen density for their maximum values projected on the galactic plane provided by Oort, Kerr and Westerhout(1958) is employed. This map was enlarged to about 4.65 times, on which the numbers of the hydrogen atoms τ'_H are counted. For 40 cepheids situated in the sector of $l=60°\sim 215°$, we obtain the following reduction formula by the method of least squares:

$$\tau_H = \underset{\pm.07}{0.14} + \underset{\pm.10}{0.63}\ \tau'_H \tag{7}$$

Employing the data of colour excess by Walraven, Muller and Oosterhoff, for all 160 cepheids of population I in the present sector, we get the correlation coefficients:

$$\tau_{Er} = 0.35 \pm 0.08 \qquad \text{for} \quad R < 8.2 \text{ kpc},$$

$$\gamma_{Er} = 0.58 \pm 0.11 \qquad \text{for} \quad R > 8.2 \text{ kpc}.$$

The smaller value for $R<8.2$ kpc may partly due to the low colour excess in Carina region. In Figs. 2 and 3, are plotted the resulting E/τ_H and E/r against R, both showing gradual decreases outwardly. This trend appears in the region $l=60°\sim215°$; in Fig. 1, the stars lying at the upper-left part have the large values of R, while the lower-right stars are situated nearer to the centre of the galaxy, though they show a correlation on the whole. Therefore, it may be meaningless to evaluate the correlation coefficient between E and τ_H for the long interval in the galactocentric distance.

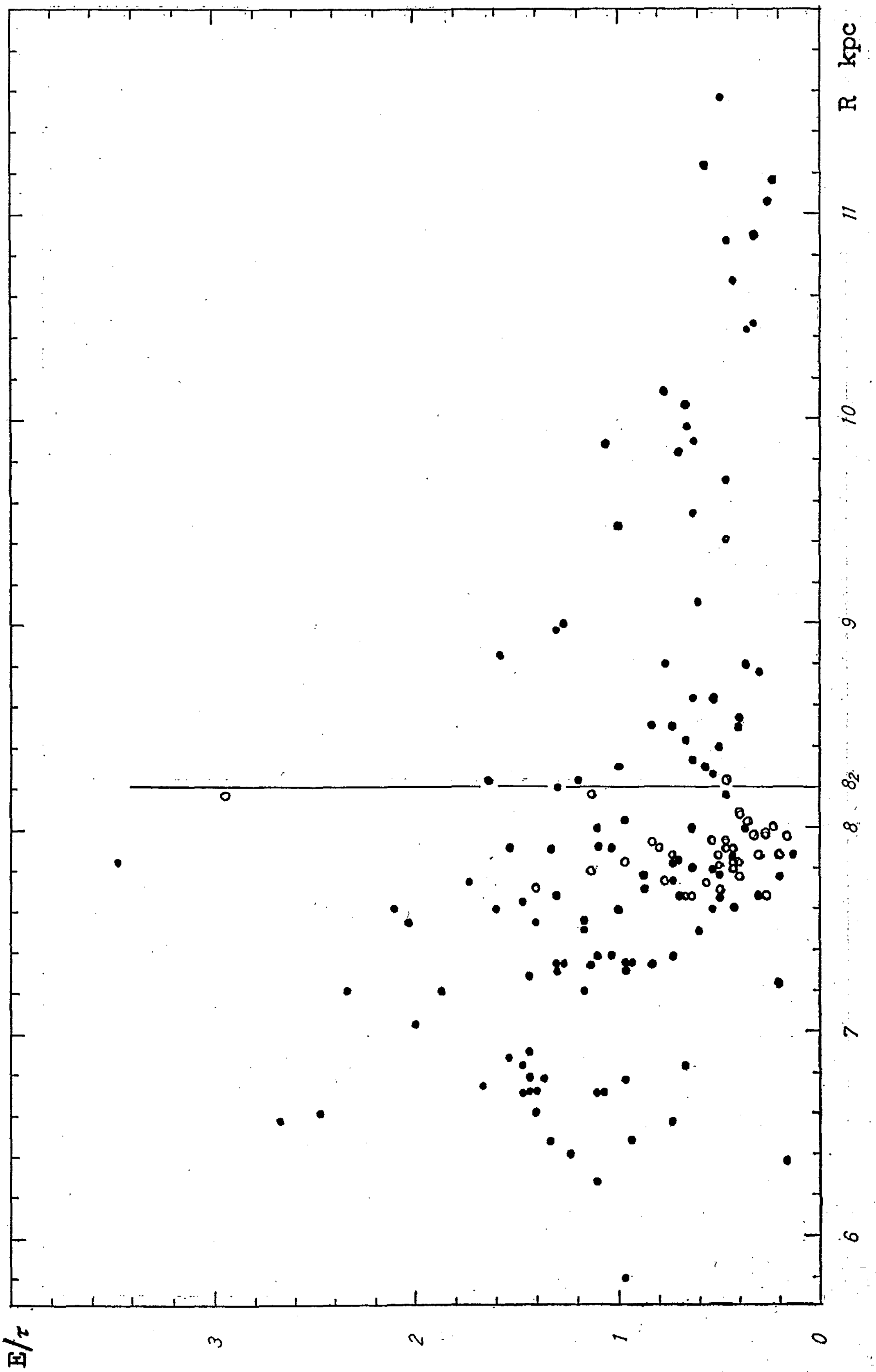

Fig. 2 Colour Excess and the Galactocentric Distance (I).

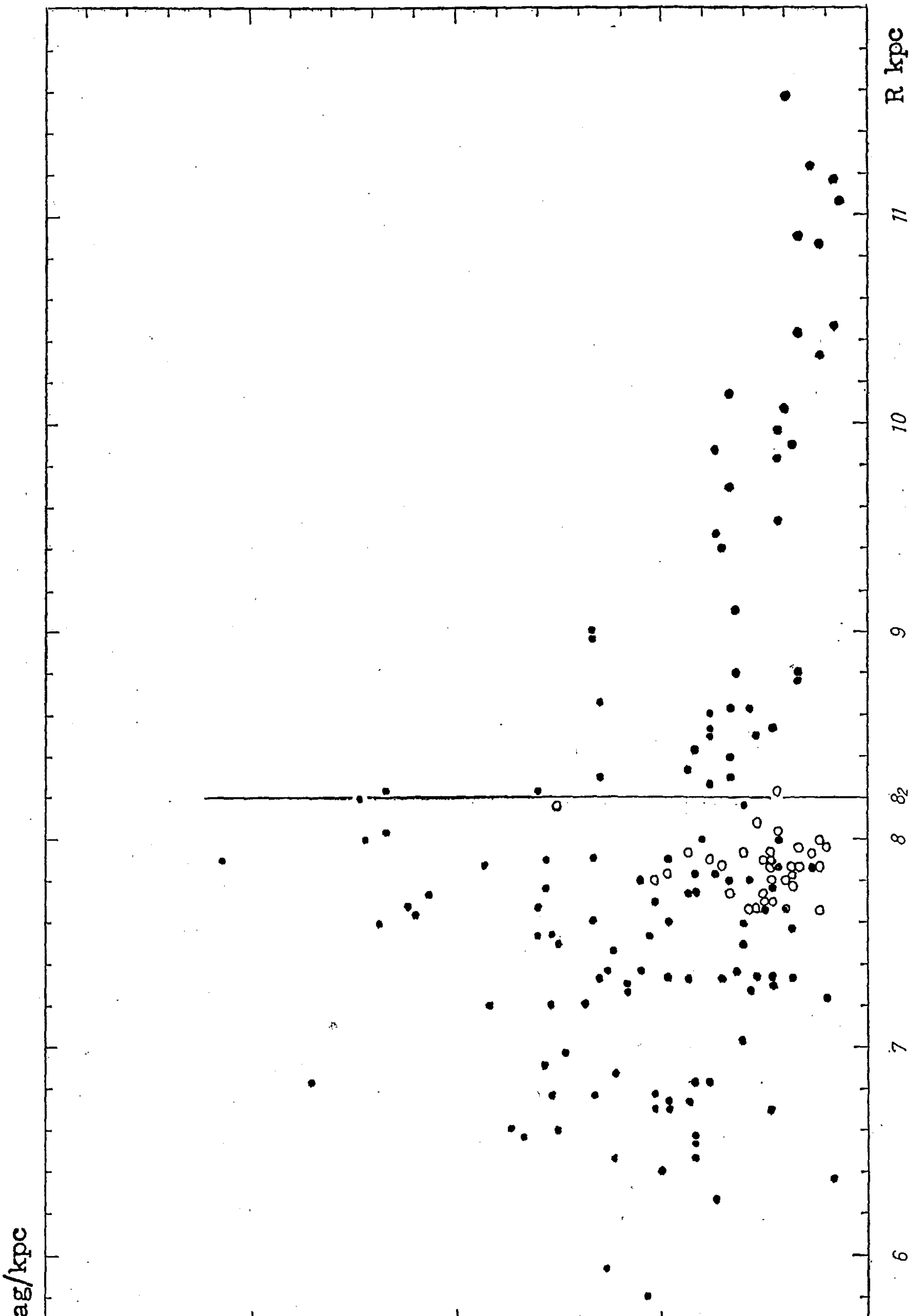

Fig. 3 Colour Excess and the Galactocentric Distance (II).

Let e be the colour excess per unit distance, here 1 kpc, then the observed colour excess E is

$$E = \int e\,dr. \tag{8}$$

If we assume that e changes linearly with R, i.e.

$$e = a + b\,R, \tag{9}$$

R is related to r through λ, the difference in longitudes of the star, l, and of the direction of the centre of the galaxy, l_0, by

$$R^2 = R_O{}^2 + r^2 - 2\,r\,R_O\,\cos\lambda,$$

R_0 being the distance of the sun from the galactic centre. Then, (8) becomes

$$E = \int_0^r \{a + b\,R\,(r, \lambda)\}\,dr$$

$$= ar + \frac{b}{2}\left\{(r-R_0\cos\lambda)R + R_0{}^2\cos\lambda + \frac{1}{R_0{}^2\sin^2\lambda}\ln\frac{R+r-R_0\cos\lambda}{R(1-\cos\lambda)}\right\}. \tag{10}$$

The least squares solutions of this equation yield for 160 stars:

$$e = 1.34 - 0.130\,R \quad \text{mag kpc}^{-1} \tag{11}$$
$$\pm 12 \qquad \pm 15$$

In this calculation, we take $l_0 = 327°.7$ and weights are assigned for E to be 0.5 to the stars which are remarked as (:) in the original paper.

The relation (11) gives

$$e_0 = 0.27 \quad \text{mag kpc}^{-1} \quad \text{at } R_0 = 8.2 \text{ kpc}, \tag{12}$$

or

$$e_{0(B-V)} = 0.25 \text{ mag kpc}^{-1} \text{ at } R_0 = 8.2 \text{ kpc}, \tag{13}$$

in *UBV* system.

4. Interstellar Grains and Colour Excess.

In 1949 van de Hulst made an elaborate work on the properties of interstellar grains to explain the space absorption, following the theories on optics of spherical particles by himself(1946) and on the growth of grains by Oort and himself(1946). We shall follow them for the discussion of the results obtained in the last section.

According to van de Hulst, number of the particles of radii between r and $r+dr$ contained in 1 cm^3 is $Cn(r/r_1)\,dr/r_1$, where r_1 is arbitrary scale factor. Introducing the definite integrals

$$A_p = \int_0^\infty n(u)\,u^p\,du, \tag{14}$$

we express,

total number of particles (cm^{-3})

$$N = C\,A_0, \tag{15}$$

total cross section (cm^2 cm^{-3})

$$\sigma = C\,A_2\,\pi\,r_1^2, \tag{16}$$

total density in space (gr cm^{-3})

$$\rho_s = C\, A_3\, \frac{4}{3}\, \pi\, s\, r_1^3, \tag{17}$$

where s is density of each particle. Hence, the interstellar extinction coefficient becomes:

$$\alpha = 1.05 \times 10^{22}\, C\, A_2\, r_1^2\, Q(\rho_1) \quad \text{mag kpc}^{-1}, \tag{18}$$

where Q is the efficiency factor for extinction, i.e. the ratio of the flux lost to the beam by scattering and absorption to the flux that is geometrically obstructed, and

$$\rho_1 = 4\, \pi\, r_1\, (m - 1)/\lambda, \tag{19}$$

being m the refractive index of the particle material.

In van de Hulst's(1949) model (g) of the theoretical distribution of the grain sizes, new nuclei of grains are formed at a constant rate through accretions of atoms and molecules, and the radii of all these nuclei are grown with a constant speed, and evaporations caused by the collisions with other particles limit the growth of the particles. For this model, values of A_p's become

$$A_0 = 0.62, \quad A_2 = 0.118, \text{ and } A_3 = 0.077, \tag{20}$$

for the dielectric particles of $m = 4/3$ which may be a reasonable value for interstellar grains. The scale factor r_1 is given by:

$$r_1^5 = \dot{r} / k\, n(0), \tag{21}$$

where $\dot{r}$ is the growth of the radius per unit of time and k measures the probability of evaporation.

In case of the colour extinction we have from (18):

$$e = \alpha - \alpha' = 1.05 \times 10^{22}\, C\, A_2\, r_1^2 \{Q(\rho_1) - Q(\rho_1'\} \quad \text{mag kpc}^{-1} \tag{22}$$

between two effective wavelengths λ and λ'. Then, by (20)

$$e = 1.24 \times 10^{21}\, C\, J, \tag{23}$$

with

$$J = r_1^2 \left\{Q(\rho_1) - Q(\rho_1')\right\} \tag{24}$$

By means of Table 2, which provides the values of $Q(\rho_1)$ for the mixture of the particles with different sizes, of van de Hulst's paper, values of J are computed and is shown in Table 1 and in Fig. 4, in which we can see that the colour excess is very

TABLE 1.

COLOUR EXTINCTION FACTOR, J.

r_1 10^{-5} cm	J 10^{-8}
0.0	+ 0.00
0.5	0.00
1.0	0.00
1.5	0.04
2.0	0.10
2.5	0.26
3.0	0.46
3.5	0.72
4.0	0.96
4.5	1.20
5.0	+ 1.33
5.5	1.30
6.0	1.12
6.5	0.76
7.0	+ 0.29
7.5	− 0.34
8.0	− 1.02
8.5	
9.0	
9.5	
10.0	

sensitive to the value of the scale factor r_1.

Therefore, the difference in colour excess is not necessarily attributed to the difference in number or density of grains simply.

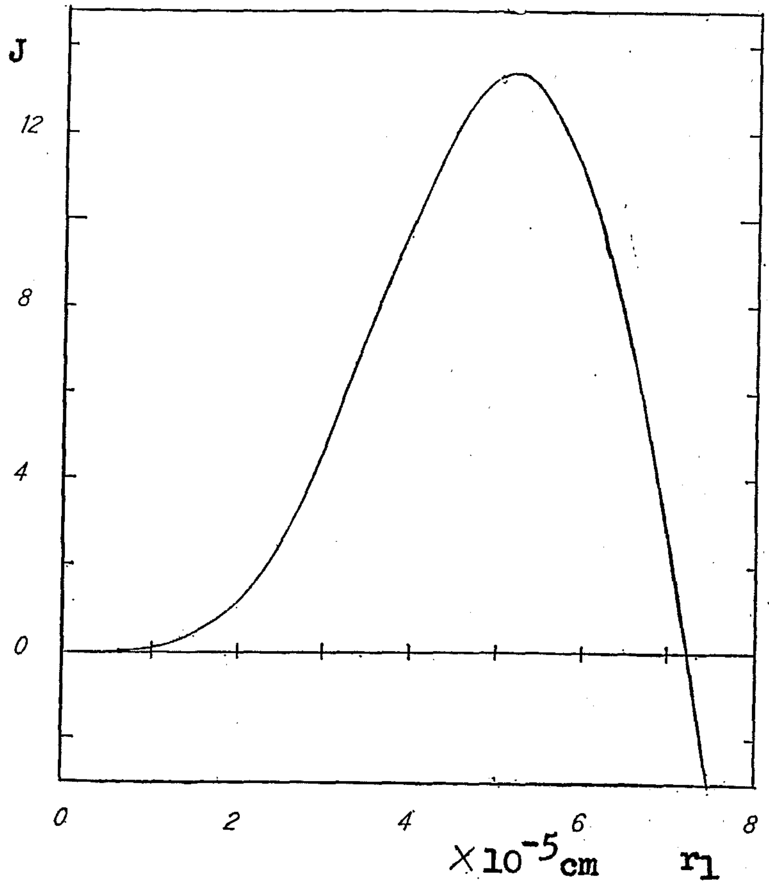

Fig. 4 Colour Extinction Factor J.

TABLE 2. EVALUATION OF C

	ρ_1			$Q(\rho_1)$			C^{-1}		
r_1	U	B	V	U	B	V	U	B	V
10^{-5} cm	10^{-5} cm	10^{-5} cm	10^{-5} cm				10^{-11}	10^{-11}	10^{-11}
1	1.20	0.95	0.76	0.21	0.10	0.06	0.2	0.1	0.1
2	2.40	1.90	1.52	1.10	0.68	0.58	2.0	2.7	3.0
3	3.59	2.86	2.28	2.15	1.52	1.00	16.2	13.6	11.7
4	4.79	3.81	3.05	2.86	2.33	1.69	38.2	37.0	35.3
5	5.98	4.76	3.81	3.10	2.85	2.33	64.7	70.7	76.0
6	7.18	5.71	4.57	3.01	3.08	2.77	90.5	110.0	130.1
7	8.38	6.66	5.33	2.80	3.08	3.02	114.6	149.7	193.1
8	9.57	7.62	6.09	2.63	2.94	3.10	140.5	186.7	258.9

5. Scale Factors of the Grain Radii.

In the last section we saw that the scale factor of the radius distribution of grains plays an important role in the colour extinction, and we shall look for any reasonable value of this factor.

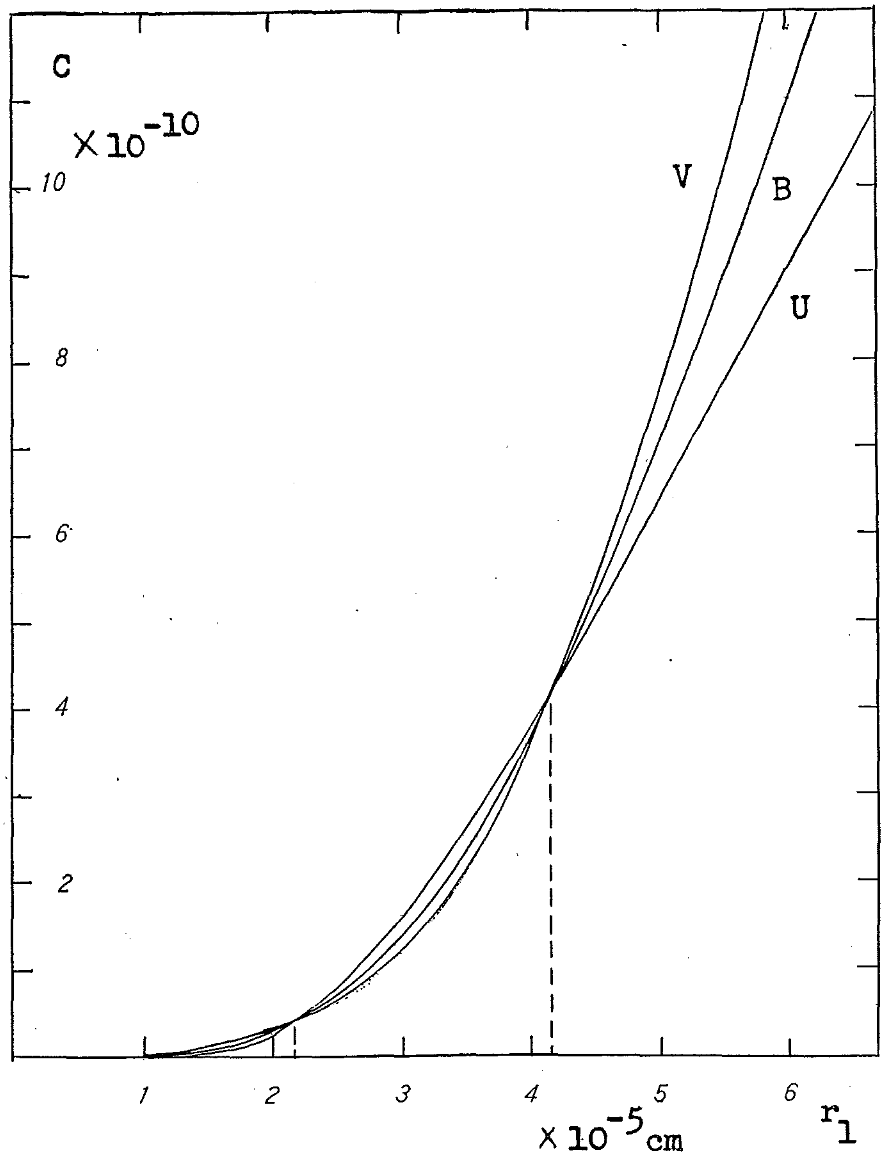

Fig. 5 Evaluation of C.

Substituting (20) into (18), we have

$$C = \frac{\alpha}{1.24 \times 10^{21}} \cdot \frac{1}{r_1^2\, Q(\rho_1)}. \tag{25}$$

Since the magnitudes of the scale factor r_1 may be of the order of visible wave length, we can evaluate C if we know the extinction for different wave lengths. In Table 2, this evaluation is shown; here, Johnson's(1951) values of absorption for *UBV* system are used, i.e.

	U	*B*	*V*
λ (10^{-5} cm)	3.5	4.4	5.5
α (mag kpc^{-1})	1.45	1.25	0.95

Since C is independent on the wave length, we obtain from Fig. 5, in which C^{-1} is plotted against r_1, two sets of C and r_1:

$$r_1 = 2.2 \times 10^{-5} \text{ cm}, \qquad C = 21 \times 10^{-13},$$

$$r_1 = 4.2 \times 10^{-5} \text{ cm}, \qquad C = 2.3 \times 10^{-13}.$$

At these two points ($C^{-1} \sim r_1$) curves for three wave lengths meet together enough accurately as shown in Fig. 5.

The adopted Johnson's extinction coefficients yield the colour extinction:

$$E_{B-V} = 0.30 \text{ mag kpc}^{-1}. \tag{26}$$

There is slight difference of 0.05 mag kpc^{-1} between two observations (13) and (26), which partly due to the fact that Johnson's value does not refer the absorption at $R_0 = 8.2$ kpc strictly. Since we have no information on standard absorption coefficients at two wave lengths in *SCI* system, we shall neglect this difference, and assume the linear formula:

$$e_{B-V} = 1.37 - 0.130\, R \quad \text{mag kpc}^{-1}, \tag{27}$$

which gives

$$e_{0(B-V)} = 0.30 \text{ mag kpc}^{-1} \text{ at } R_0 = 8.2 \text{ kpc}.$$

This assumption may not cause any serious error in the following estimations.

Next, we suppose that r_1 varies linearly with R, i.e.

$$r_1 = a_1 + b_1 (R - R_0), \tag{28}$$

and take

$$a_1 = 4.2 \times 10^{-5} \text{ cm},$$

$$b_1 = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 \times 10^{-5} \text{ cm kpc}^{-1},$$

designating models (1), (2),....., (6) of scale factor. Then, we can evaluate N, σ and ρ_S by (15), (16) and (17) through (19) by means of assumed relation (27). These values are shown in Table 3 and in Fig 6 are shown the variations of density distributions against R for six models.

In any of these six models of r_1, observed values of colour excess which is represented by (27) can be fitted. Here, we have to return to (21), van de Hulst and Oort's expression of r_1, which contains three uncertain parameters; $\dot{r}_1$ is the rate of growth of grain radius, which depends on the interstellar density of the gases other than hydrogen and helium, on their temperatures, and on the composition of the solid particles. $n(0)$ is determined by the rapidity by which new solid particles are being formed. And k, measuring the probability of evaporation, depends on the size, density and speed of the grains as well as those of interstellar clouds. Therefore, it is possible to fit any observed value of colour excesses by adjusting these quantities. We try to explain observation by changing parameters as small as possible, remaining the other parameters unchanged.

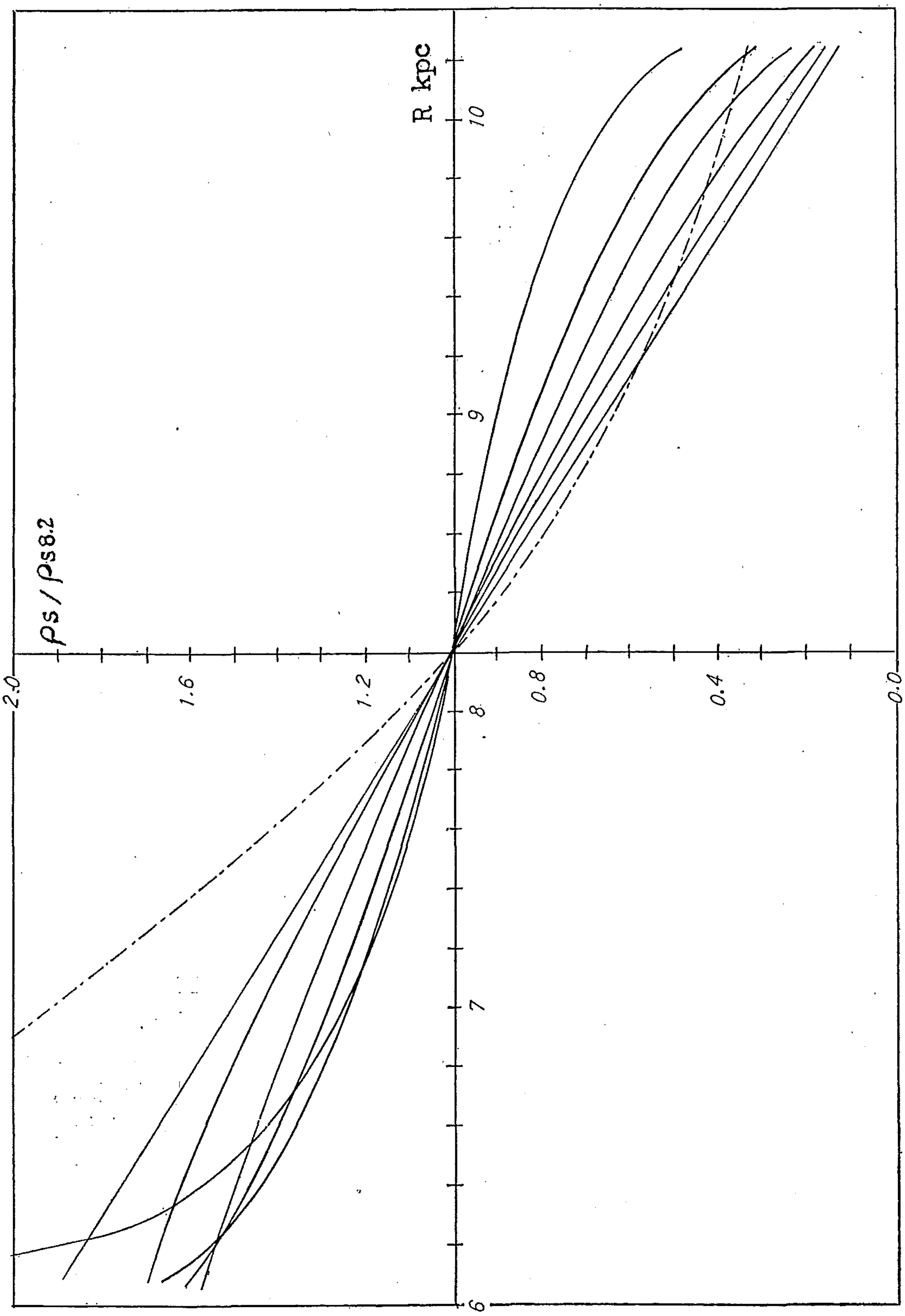

Fig. 6 Grain Density against the Galactocentric Distance.
Six curves, from the bottom in the fourth quadrant, correspond to the six models (1), (2), ..., (6) of scale factor, respectively. Chain curve denotes Schmidt's model.

TABLE 3. PROPERTIES OF GRAINS FOR 6 MODELS OF SCALE FACTOR.

Model	R	b_1	r_1	J	C	ρ_s	N	σ
	kpc	10^{-5}	cm 10^{-5}	10^{-9}	10^{-13}	g cm^{-3} 10^{-26}	cm^{-3} 10^{-13}	cm^2 cm^{-3} $\times 10^{-22}$
(1)	6.2	0.0	4.2	1.05	4.30	1.03	2.67	2.81
	7.2		4.2	1.05	3.30	.79	2.05	2.15
	8.2		4.2	1.05	2.30	.55	1.43	1.50
	9.2		4.2	1.05	1.30	.31	.81	.85
	10.2		4.2	1.05	0.30	.07	.19	.20
(2)	6.2	0.2	3.8	0.86	5.26	0.93	3.26	2.81
	7.2		4.0	0.96	3.62	0.75	2.24	2.15
	8.2		4.2	1.05	2.30	0.55	1.43	1.50
	9.2		4.4	1.15	1.19	0.33	0.74	0.85
	10.2		4.6	1.24	0.26	0.08	0.16	0.20
(3)	6.2	0.4	3.4	0.66	6.85	0.86	4.25	2.95
	7.8		3.8	0.86	4.04	0.72	2.50	2.16
	8.2		4.2	1.05	2.30	0.55	1.43	1.50
	9.2		4.6	1.24	1.11	0.35	0.69	0.87
	10.2		5.0	1.33	0.24	0.10	0.15	0.22
(4)	6.2	0.6	3.0	0.46	9.83	0.86	6.09	3.28
	7.2		3.6	0.76	4.57	0.69	2.83	2.20
	8.2		4.2	1.05	2.30	0.55	1.43	1.50
	9.2		4.8	1.30	1.05	0.38	0.65	0.90
	10.2		5.4	1.32	0.24	0.12	0.15	0.26
(5)	6.2	0.8	2.6	0.30	15.06	0.87	9.33	3.80
	7.2		3.4	0.66	5.26	0.66	3.26	2.26
	8.2		4.2	1.05	2.30	0.55	1.43	1.50
	9.2		5.0	1.33	1.03	0.42	0.64	0.96
	10.2		5.8	1.21	0.26	0.16	0.16	0.32
(6)	6.2	1.0	2.2	0.15	30.11	1.07	18.67	5.36
	7.2		3.2	0.56	6.20	0.66	3.84	2.35
	8.2		4.2	1.05	2.30	0.55	1.43	1.50
	9.2		5.2	1.33	1.03	0.47	0.64	1.03
	10.2		6.2	1.97	0.33	0.25	0.20	0.47

We suppose that both $\dot{r}$ and $n(0)$ depend only on gas dendity, then, r_1^5 varies with the chance of the evaporations, which further, supposed to depend only on the total cross sections of the grains, that is:

$$r_1\ \sigma^{1/5} = \text{const.} \tag{29}$$

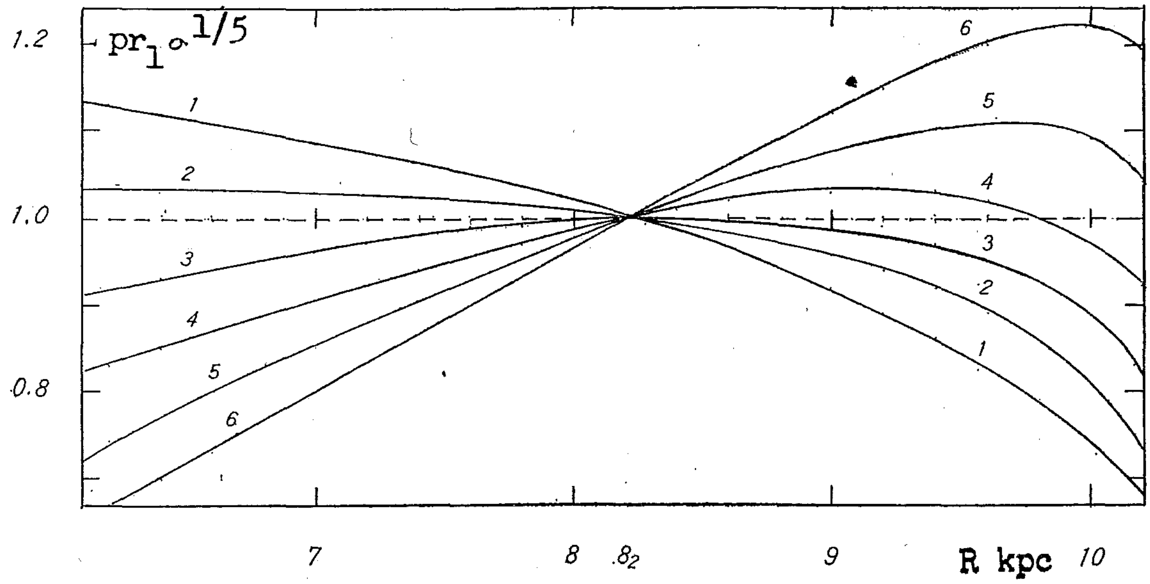

Fig. 7 Values of $pr_1\sigma^{1/5}$ for 6 Models.

In Fig. 7, $pr_1\sigma^{1/5}$ are shown for six models, p being constants so as to make $pr_1\sigma^{1/5}$ to be equal to unity at $R_0=8.2$ kpc. Under the condition (29), we select the values of r_1 from Fig. 7, as follows:

R	6.2	7.2	8.2	9.2	10.2 kpc
r_1	3.71	3.92	4.20	4.65	5.67 × 10^{-5} cm.

6. Final Results.

We can now evaluate the properties of grain distribution easily, which are contained in Table 4.

TABLE 4. PROPERTIES OF GRAIN DISTRIBUTION

R	r_1	J	ρ_s	N	σ
kpc	10^{-5} cm	10^{-9}	10^{-26}. gr.cm^{-3}	10^{-13}. cm^{-3}	10^{-22}. cm^2. cm^{-3}
6.2	3.71	0.82	0.91	3.42	2.82
7.2	3.92	0.92	0.73	2.34	2.16
8.2	4.20	1.06	0.55	1.43	1.50
9.2	4.65	1.25	0.36	0.68	0.88
10.2	5.67	1.26	0.15	0.16	0.30

The density, number and cross section of grains decrease together outwardly. However, this fact cannot be connected to the observed decrease of colour excess, but the scale factor of grain sizes, or mean size of grains must be taken into account. At about $R=9.8$ kpc, $r_1=5.2\times10^{-5}$ cm, colour excess factor J takes its maximum value. Hence, outward decrease of the grain size gives negative effect to the decrease of the colour excess for $R<9.8$ kpc, and for $R > 9.8$ kpc colour excess decrease excessively.

In fact, it is unreasonable to attribute difference in the colour excess to the difference in the number of the grains, assuming the grain size distribution to be unchanged. In the region of lower density of the grains, the occurences of collision between the grains, and hence the chances of evaporation are also small, causing the overgrowth of the grains by accretion of atoms or molecules. This overgrowth changes the value of J, making the grains more effective for selective absorption, or less effective. This is the situation what appears in Table 4.

The density distribution of Schmidt's(1956) model on the galactic plane is shown in Fig. 6 by dotted curve, which shows similar trend to our final model outside the sun, crossing just on the same point at $R=10.2$ kpc. This fact may indicate that our estimation of grain distribution is not so unreasonable. For $R<8.2$ kpc, Schmidt's model departs largely from our result. This may partly due to the fact that, in this region, there exists fairly wide areas of low hydrogen density, as is recognized from the map of hydrogen distribution on the galactic plane, and hence, that grain density may also lower in comparison with the general tendency of mass distribution over the galactic plane.

7. Conclusions.

The correlation coefficient between colour excess and the number of neutral hydrogen atoms is likely to be 70 per cent or larger for the sector between galactic longitutes $l=60\sim215°$. On the whole, colour excess per unit distance seems to decrease outwardly on the galactic plane, and is expressed:

$$
\begin{aligned}
e = &\ 1.37 - 0.130\,R \quad \text{mag kpc}^{-1} \\
&\pm 12 \quad \pm 15
\end{aligned}
$$

in UBV system.

Under the assumption that the probability of the evaporations of the grains depends only on the total cross section of the grains, mean size of the grains increases outwardly, causing the linear decrease of grain density. This increase may due to the lower density, which is favourable for grains to grow excessively, in the outer regions of the galactic plane.

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