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A GEOID OF THE IZU ISLANDS AREA

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Abstract

Two kinds of geoid map have been drawn for the area of the Izu Islands by employing the data of over sea levelling among the islands and of vertical deflection at each island. The over sea levelling has been made since 1967, and two closed levelling loops have been constructed among the islands and the Izu Peninsula, joinning at Nii Sima to each other. Closure error of each loop is within 20—30cm, when geoid undulation inside of each island and vertical deflection are taken into consideration. A geoid map thus drawn on the Tokyo Datum indicates a systematic shift of the Tokyo Datum. Another geoid map on a global datum, SAO-SE3, shows effects of the gravity low over the trench lying to the east of the mainland of Japan and the gravity high over the southern area of the Izu Islands. Extention of the geoid undulation from the mainland of Japan (Ganeko, 1976) to this area seems satisfactory.

Key words: geoid——over sea levelling.

1. Introduction.

Over sea levelling by the trigonometric method have been made by the Hydrographic Department since 1967 at the area of the Izu Islands which is one of the active regions of earthquake. Purpose of this project is to obtain basic data for the earthquake prediction.

Observations have been performed at a rate of one pair islands (or mainland) per annum. A view of relative positions of the islands connected by the over sea levelling is shown in Figure 1. Observation year is attached to each connecting line.

At each of pair islands, two observation sites have been located with a short distance but an adequate height difference. Zenith distances of two sites at the confronted island have been measured mutually from each site at about same time. As shown in Figure 2, observation data of eight zenith distances are thus obtained every time. We shall call them as one *set* of data. Measurements have been made by theodolites: Wild-T3, and-T2 and Kern-DKM3A and-DKM3. Individual observation data obtained from 1967 to 1975 with fuller description about observation method are reported by Ganeko and Sasaki (1977).

Except for the heights for Kōzu Sima and Sirahama, geodetic positions of the sites are referred to those of the nearby national triangulation points. Geodetic coordinates of the triangulation points have been taken from the "Resulting Table" of the Geographical Survey Institute (GSI). In general the list contains the coordi-

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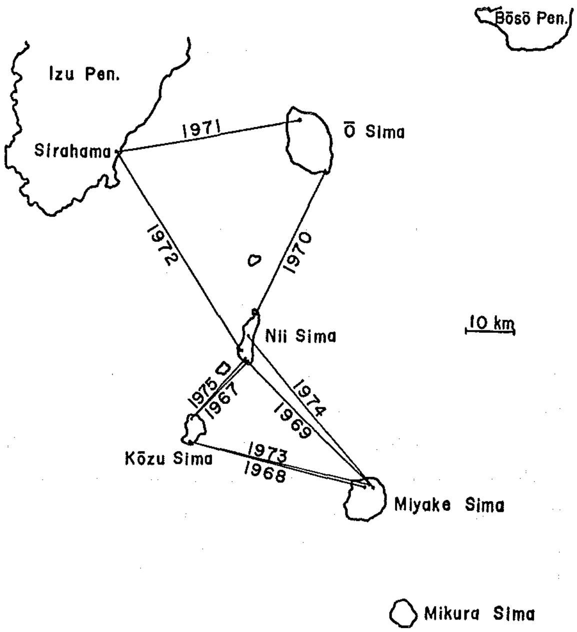


Fig. 1 Networks of over sea levelling for the Izu Islands area.

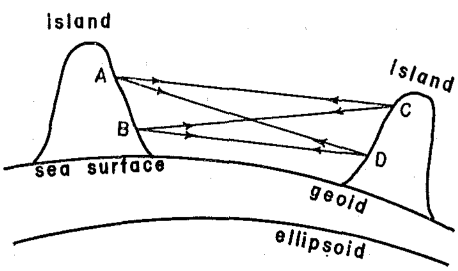


Fig. 2 Over sea levelling observation.

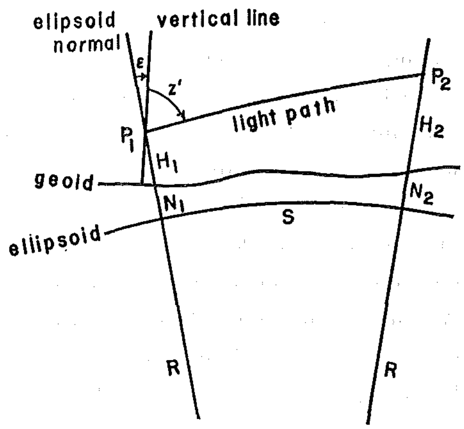


Fig. 3 Geometry of trigonometric levelling. Ellipsoidal height $h=H+N$

nates of the triangulation points referred to the Tokyo Datum. However, for the triangulation points in Nii Sima, Kōzu Sima and Miyake Sima, to which the 3rd order triangulation net of the mainland has not been connected, the Resulting Table gives the height determined independently at each island from its local mean sea level. During the period of the over sea levelling observation, heights of the observation sites in Kōzu Sima were connected ditectly to the bench mark (4.88m) at Kōzu Sima Tide Station. Heights of the sites at Sirahama were connected directly to the bench mark Itiki of the national levelling net.

Besides the over sea levelling, observations of astronomic position have been made over the Izu Islands, including most of the levelling sites, by means of the constant altitude method. Vertical deflection is obtained as the difference between astronomic and geodetic positions for each site. Devices employed are Tsubokawa photoelectric astrolabe, Carl Zeiss Ni-2 astrolabe with a 60°-prism and Kern-DKM3A with high sensitive levels and an impersonal micrometer. Individual data with preliminary reduction of these observations have been reported by Suzuki and Harada (1966), Suzuki and Sugimoto (1967), Sugimoto and Harada (1968), Takemura and Koyama (1970) and Sasaki and Kanazawa (1977).

In the following the geoid undulations over the Izu Islands area are calculated using the above two kinds of observation data.

2. Geoid-height difference by over sea levellings.

We suppose that the zenith distance z' of a point P_2 is measured at another point P_1. See Figure 3. Let

h_1 and h_2 : ellipsoidal heights of P_1 and P_2,
α : azimuth of P_2 at P_1,
ξ and η : vertical deflection components at P_1 along meridian and prime vertical,
ε : vertical deflection component at P_1 in the direction of $\overrightarrow{P_1P_2}$,
s : ellipsoidal distance between P_1 and P_2,
R : radius of mean curvature of the ellipsoidal arc between P_1 and P_2,
K : refraction coefficient.

α, s and R are calculated from the geodetic coordinates of P_1 and P_2. We can calculate ε from ξ and η through the well known relation

$$\varepsilon = \xi \cos \alpha + \eta \sin \alpha.$$

The observed zenith distance refers to the local plumb line, with which the ellipsoidal zenith distance z at P_1 is related by

$$z = z' + \varepsilon.$$

Hence, the difference in the ellipsoidal height between P_1 and P_2 can be calculated from z by the formula

$$h_2 - h_1 = s \cot z + \frac{s^2}{2R} - \frac{s^2}{2R} K. \tag{1}$$

The second term of the righthand side of (1) corresponds to the curvature of the ellipsoid with sufficient accuracy in our case, because the ellipsoid heights of P_1 and

P_2 are far smaller than the length of R and the zenith distance is very close to 90°. The third term is a correction for the curvature of the light pass, assuming it to be a part of circle. The effects of this assumption will reflect in the various errors estimated in the procedure of data reduction.

The ellipsoidal height is the sum of the height above mean sea level (above the geoid) H and the geoid height N, namely

$$h=H+N. \tag{2}$$

From (1) and (2) we obtain the difference in geoid height

$$\Delta N = N_2-N_1=-(H_2-H_1)+s\cot z+\frac{s^2}{2R}-\frac{s^2}{2R}K. \tag{3}$$

Since ε is a small quantity, we can rewrite (3) as

$$\Delta N=-(H_2-H_1)+s\cot z'-s\varepsilon+\frac{s^2}{2R}-\frac{s^2}{2R}K, \tag{4}$$

so that the effect of the vertical deflection appears explicitly. The magnitude of the last term of (4) amounts to about 10 m in our case, say, for $s=30$ km for example. Actually K is not a constant but a variable quantity depending on atmospheric condition and hence changes with time. Its value varies in a range of 0.13~0.15 usually.

In principle it is possible to evaluate K using the meteorological data. But such a procedure have great difficulties practically, especially in gathering the data of vertical gradient of air temperature. Therefore, it is more practical to consider (4) as an observation equation with two unknowns ΔN and K. It is remarked here that the geoid-height difference between the two sites in the same island is negligibly small because they are located enough closely to each other. Hence only the two unknowns above are reasonably adopted to be solved through observation equations for each observation set.

We can figure out other observation equations with more unknowns taking the effect of height dependence of the refraction into consideration. There sometimes happened also that the apparent zenith distance changed suddenly at one site solely; this fact may suggest the air blocks of small size take an important part in the large change of observed zenith distance. If this would be true, an expression of the refraction coefficient by a simple function of height would have no advantage but would increase the mean errors of ΔN and K calculated by (4). However, since the purpose of the present paper is to get a view of the geoid undulations around the Izu Islands area, we shall not make further investigations on the refraction effect for the present.

Result of the least squares calculation of (4) is given in Table 1 and discussed in Section 4.

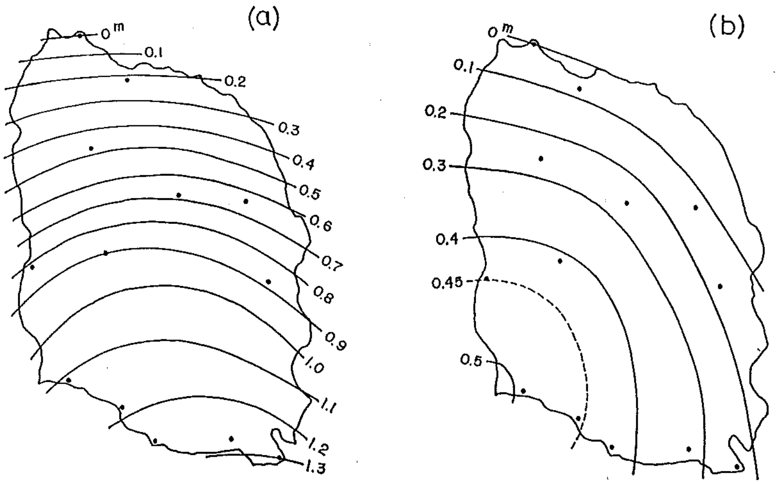


Fig. 4 Astrogeodetic geoid of Ō Sima.

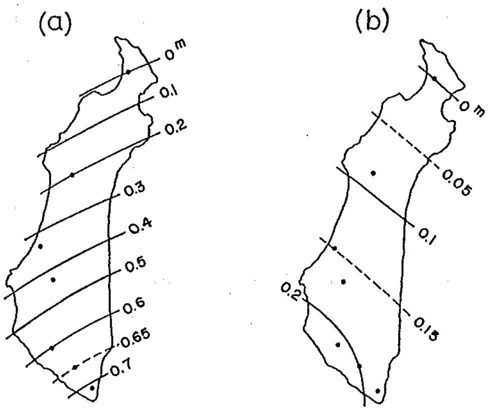


Fig. 5 Astrogeodetic geoid of Nii Sima.

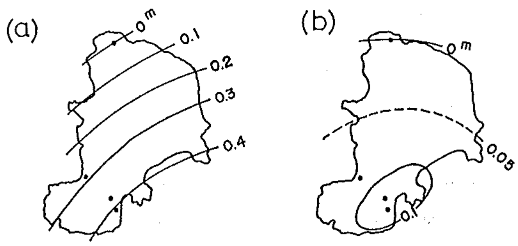
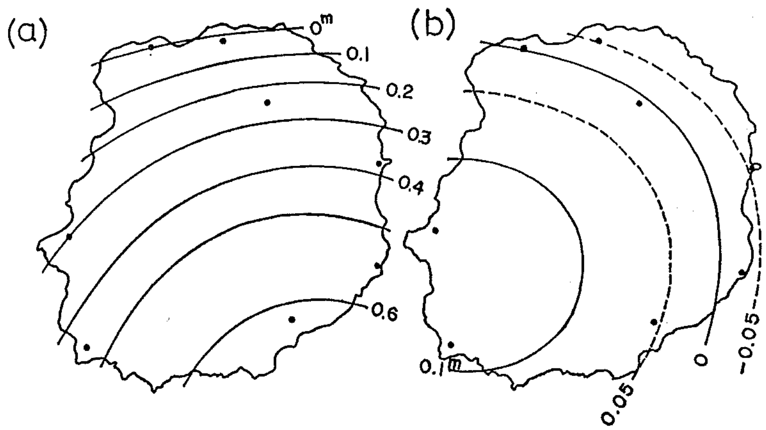


Fig. 6 Astrogeodetic geoid of Kōzu Sima.



(a) Tokyo Datum (b) SAO-SE3 system

Fig. 7 Astrogeodetic geoid of Miyake Sima.

3. Inland geiod of the Islands.

The geoid undulations inside of an island are computed by using the Helmert's formula

$$N_q - N_p = -\int_P^Q (\xi \cos A + \eta \sin A)\ ds, \tag{5}$$

where N_p and N_q are the geoid heights at points P and Q ; ξ, η are components of the vertical deflection in the conventional expression ; A is azimuth of tangential line of the integral path from P to Q. If the distance between them is not so large, the integral is approximatad by

$$N_q - N_p = -(\varepsilon_p + \varepsilon_q) s/2, \tag{6}$$
$$\varepsilon_p = \xi_p \cos A + \eta_p \sin A,$$
$$\varepsilon_q = \xi_q \cos A + \eta_q \sin A,$$

where A is the azimuth of the line $\overrightarrow{PQ}$. (6) can be applied effectively to the case in which data of vertical deflection are available only at P and Q.

The geoid undulations on the Tokyo Datum inside each island are calculated by using (6) for Ō Sima, Nii Sima, Kōzu Sima and Miyake Sima, and they are shown in Figures 4 (a), 5 (a), 6 (a) and 7 (a), where one of the deflection stations in the north part of each island is taken as the starting point ot the integration. Locations of the deflection stations are indicated by dots in the figures. Accuracies of the geoid undulations presented here are estimated to be several centimeters for Ō Sima and about 10 cm for the other islands. We see the systematic effect due to the adoption of the Tokyo Datum in the Figures (a)'s.

In order to eliminate this effect, the following datum shift is applied to reduce geocentric rectangular coordinates to one of the world geodetic system, SAO-SE3 (Gaposchkin et al., 1973) :

$$DX = -136\ \text{m},\ DY = +541\ \text{m},\ DZ = +681\ \text{m}. \tag{7}$$

Using (7) we can calculate the distance between the reference ellipsoids of the Tokyo Datum and of the SAO-SE3 as illustrated in Figure 8. By adding this distance to the relative geoid undulations on the Tokyo Datum, we can translate the geoid undulations on to the SAO-SE3 system. They are seen in Figures 4 (b), 5 (b), 6 (b) and 7 (b). The local features of the geoid proper to each of the islands are now apparent in these figures, and it can be seen that the highest point of the geoid at each island is not situated at the centre of the island. This fact is interesting especially for Ō Sima and Miyake Sima.

It may be expected that the peak of the local geoid of Ō Sima is located at the center of the island from a view of the distribution of free-air gravity anomaly as shown in Figure 9 which has been produced by using the data reported by Yokoyama and Tajima (1957). We suppose here that the apparent shape of the geoid is produced by a superposition of a local geoid caused by mass concentration at the island and a general geoid which has a simple shape over wider area around the island. We can estimate the feature of the general geoid by applying any of suitable models, the following assumption is introduced : the local geoid has a shape of

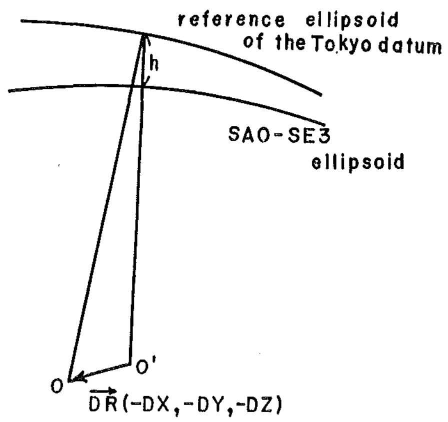


Fig. 8 Conversion of geodetic datums.

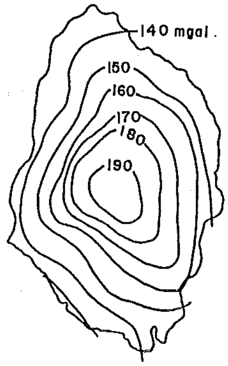


Fig. 9 Free-air gravity anomaly over Ō Sima.

rotational parabola and the general geoid is expressed by a plane : namely the vertical cross sections of them are expressed by the following formulae, respectively,

$$N_L = -as^2 + N_{LO}, \qquad a>0 \tag{8}$$

$$N_S = -bs + N_{SO}, \qquad b>0 \tag{9}$$

where N_{LO}, N_{SO} are constants and s is a parameter of distance. The shape of actual geoid is formed by the sum of (8) and (9)

$$N_A = N_L + N_S = -a\left(s + \frac{b}{2a}\right)^2 + N_{LO} + N_{SO} + \frac{b^2}{4a}.$$

Now we find that the peak of the geoid is shifted by the amount of $b/2a$ to the direction of the higher portion of slope of the general geoid. Applying this simple model to Ō Sima, we find that the island is located on a geoid slope with a gradient by 5 cm/km down to the northeast. Similarly for Miyake Sima, the gradient is estimated to be about 2 cm/km downward in the same direction.

4. Geoid around the Izu Islands.

As described in Section 1, we have horizontal coordinates on the Tokyo Datum at each levelling site, and vertical deflection data are also available for the most of the sites. For those sites where observations of astronomic position have not been made, vertical deflections are estimated from the data at the nearest astronomic sites. We can thus calculate the geoid-height difference between each island pair from the eight observation equations of the form of (4) for each observation set. Results are listed in Table 1 with some relevant data. In the 4th column are given the weighted means of the geoid-height difference with formal mean errors obtained through the least squares calculations. However, in practice, errors due to the adopted vertical deflections and to the adopted heights of the levelling sites are certainly larger than the formal errors. Error ($\delta\varepsilon$) of the observed vertical deflection is estimated to be $\pm 1\sim 2''$ depending on the method of observation.* In the 5th

Table 1 Geoid-height difference by over sea levelling

pair island A−B	observation date	distance	geoid-height difference $N_B - N_A$	m. e.	Possible error for $\delta\varepsilon = \pm 2''$	mean refraction coefficient	geoid-height difference on SAO-SE3
		km	m	m	m		m
Nii Sima - Kōzu Sima	1967. 11. 22	15. 6	+0. 83	±0. 02	±0. 15	0. 1458	+0. 72
	1975. 5. 27	15. 7	+0. 83	0. 02	0. 15	0. 1385	+0. 66
Kōzu Sima - Miyake Sima	1968. 11. 16	35. 9	+2. 00	0. 03	0. 35	0. 1462	−0. 20
	1973. 10. 23, 24, 25	37. 2	+1. 94	0. 04	0. 36	0. 1440	−0. 34
Nii Sima - Miyake Sima	1969. 11. 23	35. 3	+3. 52	0. 03	0. 34	0. 1489	+0. 84
	1974. 10. 14, 15	38. 8	+4. 03	0. 03	0. 38	0. 1438	+1. 07
Ō Sima - Nii Sima	1970. 11. 21	31. 5	+1. 54	0. 04	0. 31	0. 1407	+0. 53
Sirahama - O Sima	1971. 12. 6, 7	36. 8	−0. 05	0. 05	0. 36	0. 1455	−1. 39
Sirahama - Nii Sima	1972. 11. 14, 15	46. 5	3. 08	0. 02	0. 45	0. 1503	−0. 40

column are given values of $s \cdot \delta\varepsilon$, taking $\delta\varepsilon = \pm 2''$, as an example for estimating the effect to the geoid-height difference. As described in Section 1, heights of the levelling sites are referred to those of the triangulation points, and errors in these references may be around $\pm$20cm.

It was incidentally found through the investigation of the levelling data that the heights of the triangulation points in Kōzu Sima given in the GSI Resulting Tables are 2.8m higher than those by actual measurement from the mean sea level at the bench mark of tide station in Kōzu Sima. If these nominal values would be adopted in the calculation of the geoid-heght difference, there would appear an abnormal dent of geoid around the island. Such a dent could not be interpreted from the distribution of gravity anomaly around the Izu Islands area (Segawa and Bowin, in press). Then, we have adopted the actual height for Kōzu Sima to evaluate the geoid-height difference given in Table 1. (See addendum in proof.)

The geoid-height differences transformed into the SAO-SE3 system are also given in the last column of Table 1.

We can now view the geoid undulation around the Izu Islands in Table 1 and the geoid inside of each island in Figures 4, 5, 6 and 7. As shown in Figure 1, two loops of the levelling have been formed, i. e. Sirahama, Nii Sima, Ō Sima, Sirahama; Nii Sima, Kōzu Sima, Miyake Sima, Nii Sima. Loop closures of the geoid-height differences are within 14 cm for the former loop and about 30cm for the latter. The large misclosure for the latter loop comes mainly from the bad accuracy of the vertidal deflection data at Kōzu Sima and Miyake Sima where astronomic observations were made mostly by visual method.

A geoid contour map around the Izu Islands on the Tokyo Datum is produced by extending the astrogeodetic geoid map of Japan made by Ganeko (1976). It is presented in Figure 10, in which the geoid-height at Sirahama is taken to be zero for convenience's sake. Figure 11 is the geoid in the SAO-SE3 system, which is also drawn by extending the geoid map of the same kind by Ganeko (1976), in which the datum point of the Tokyo Datum is taken as the reference point of the geoid-height. In preparing these maps, the over sea levelling between Miyake Sima and Kōzu Sima is less weighted than the others because of the bad accuracy of the vertical deflection data as explained above. In Figure 12, the free-air gravity anomaly map by Segawa and Bowin (in press) is laid on Figure 11. We can easily understand the rapid change of the direction of geoid contour lines in the Izu Island area on seeing the existence of a vast area of gravity high to the south of Izu Islands. The gravity high may overcompensate the gravity low of the Izu-Ogasawara Trench, and contribute to a geoid rise over the south area of the Izu Islands. On the other hand, the gravity lows in Suruga Bay and off Ensyū Nada may contribute to a geoid dent there. The geoid slope suggested in the last paragraph of Section

* For example, visual observations with Carl Zeiss Ni-2 astrolabe include some amount of personal equation in longitudinal component. They are, of course, corrected in the reduction, but the resultant deflections are still considered to be not accurate enough for the author.

3 is clearly found in Figure 11 and its gradient agrees fairly well with those estimated there.

5. Concluding remarks.

Due to the restriction of mutual distances, determination of geoid height difference by means of the over sea levelling cannot be applied to the area further from Mikura Sima. Evalultuin of geoid undulation for the outer area in the Pacific Ocean has to be made by other methods ; one of them is gravimetrical one using terrestrial gravity data. A simple application of the Stokes' integral requires gravity data over the entire surface of the earth, but the combination of the terrestrial gravity data and the coefficients of the earth potential derived from the satellite tracking data enables to compute a detailed gravimetric geoid, although the terrestrial data are not available at present with sufficient density to cover the whole surface of the earth (Strange et al., 1972 : Ganeko, 1975). By applying this method, computation of a gravimetric geoid in Japan and the adjacent seas is now under preparation. The geoid obtained in the present paper will be of use for the calibration of the detailed gravimetric geoid.

The accuracy of the geoid-height at Miyake Sima is roughly estimated to be about ± 30 cm. Since positions of the levelling sites are based on those of the triangulation points, the accuracy of the height above mean sea level (above the geoid) of the triangulation points directly reflects into that of calculated geoid-height differences. Although the case of Kōzu Sima may be an unusual one, the possibility of some error still remains concerning the other islands. Further investigation on this problem and improvement in the accuracy of the vertical deflection data are needed to compute the geoid undulations more precisely.

It should be remarked that the derivation of the geoid maps in Figures 10 and 11 are based on two serious assumtions. They are (i) assumption that the adopted heights of the levelling sites are referred to the ture mean sea levels at respective islands, and (ii) assumption that the mean sea level represents the geoid surface. Ultimate sources of the heights in Nii Sima, Kōzu Sima and Miyake Sima are tide observations in 1910's, detailed records of which seem to have gotten scattered and lost. It is probable that the periods of these observations were limited in some short terms. On the other hand, the mean sea levels in the Izu Islands change significantly according as the situation of the flow axis of the Kuroshio. In particular at Kōzu Sima and Miyake Sima, monthly means of sea level change often over 50 cm within one year (e. g. Shoji, 1972). Detailed analysis of tide observation data for longer periods is desired.

Mean sea surface never coincides with the geoid surface, especially in the area where ocean current dominates. Investigation on the discrepancy between them is entangled with the analysis of tide data stated adove, and is an important problem for research in marine geodesy as well as in oceanography. Information from the gravimetric geoid may become a good tool to separate these two surfaces so that

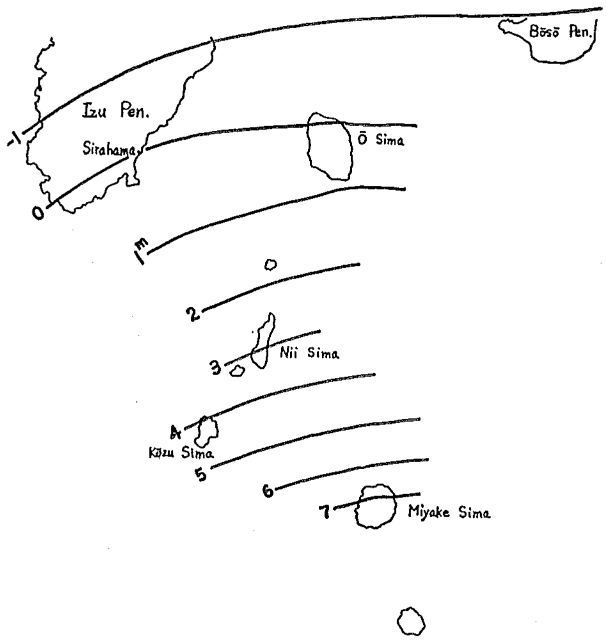


Fig. 10 Geoid contours around the Izu Islands on the Tokyo Datum. Geoid height at Sirahama is taken to be zero.

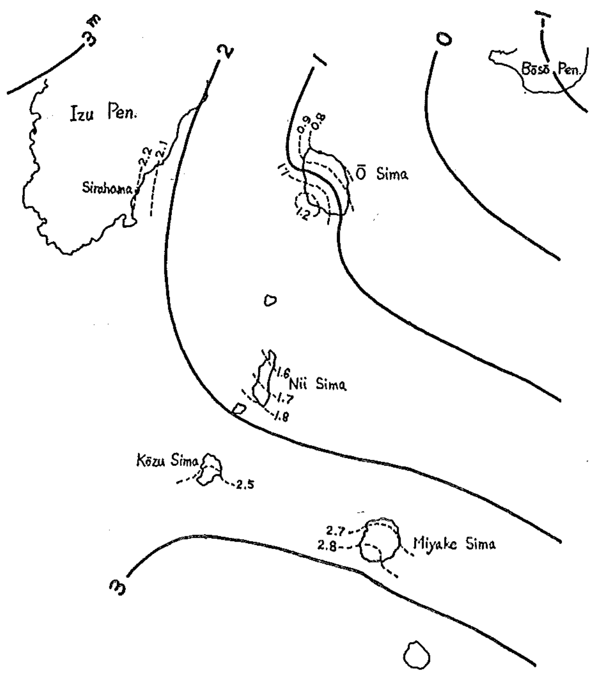


Fig. 11 Geoid contours around the Izu Islands in the SAO-SE3 system, extended from the astrogeodetic geoid map of Japan by Ganeko (1976).

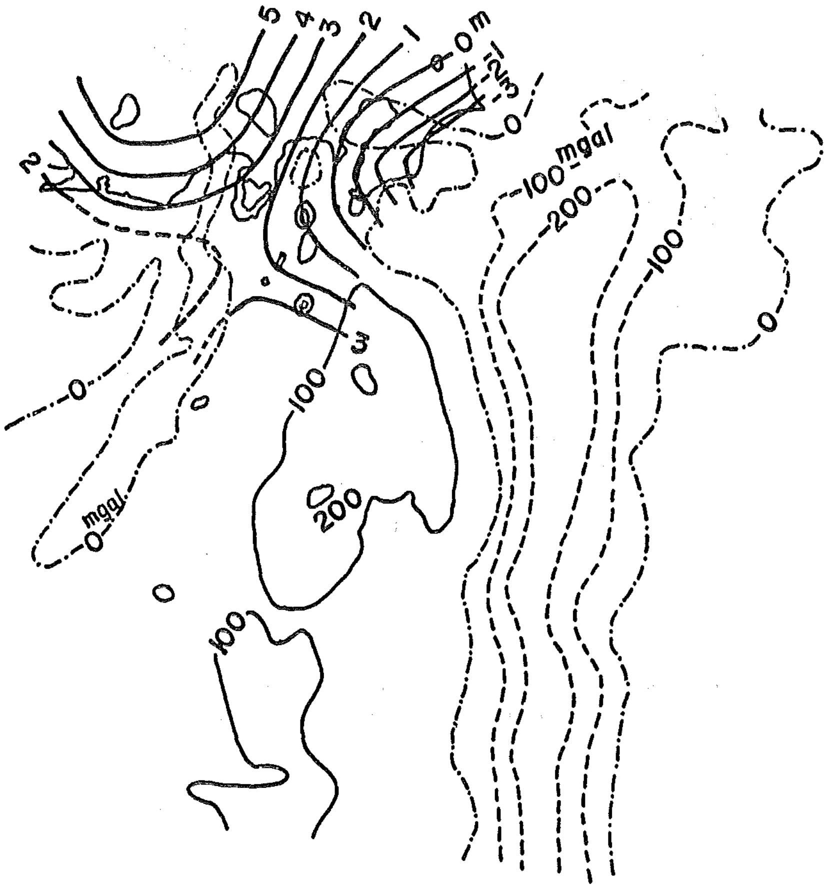


Fig. 12 Free-air gravity anomaly (Segawa and Bowin, 1976) laid on Figure 11.

dynamics of the ocean current be improved. Such improvement may be fed back to refine our knowledge on the geoid in ocean area.

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References

Ganeko, Y. 1975, *Center for Astrophysics, Harvard College Observatory and Smithsonian Astrophysical Observatory*, Preprint Series No. 399.

Ganeko, Y. 1976, *SAO Special Report* 372.

Ganeko, Y. and Sasaki, M. 1977, *Data Report of Hydrographic Observations, Series of Astronomy and Geodesy*, No. 12. p. 60.

Gaposchkin, E. M., Latimer, J. and Veis, G. 1973, *SAO Special Report* 353, p. 315.

Sasaki, M. and Kanazawa, T. 1977, *Data Report of Hydrographic Observations, Series of*

Astronomy and Geodesy, No. 12, p. 47.

Segawa, J. and Bowin, C. in press, *Jour. Phys. Earth.*

Shoji, D. 1972, The Kuroshio south of Japan in 1965 in *Researches in Hydrography and Oceanography, in commemoration of the centenary of the Hydrographic Department of Japan*, Ed. D. Shoji, Tokyo, p. 113.

Strange W. E., Vincent, S. F., Berry, R. H. and Marsh, J. G. 1972, A detailed gravimetric geoid for the United States in *The Use of Artificial Satellite for Geodesy*, Ed. S. W. Henriksen, A. Mancini and B. H. Chovitz, *AGU Monograph* **15**, Washington, D. C., p. 169.

Sugimoto, K. and Harada, Y. 1968, *Data Report of Hydrographic Observations, Series of Astronomy and Geodesy*, No. 3, p. 14.

Suzuki, H. and Harada, Y. 1966, *ibid.*, No. 1, p. 36.

Suzuki, H. and Sugimoto, K. 1967, *ibid.*, No. 2, p. 20.

Takemura, T. and Koyama, K. 1970, *ibid.*, No. 5, p. 19.

Yokoyama, I. and Tajima, H. 1957, *Bull. Earthquake Inst.*, No. 36. pt. 1, p. 23.

Addendum in proof: GSI, having found the height data for Kōzu Sima to be erroneous, is now making remeasurement at the island.