ON THE GALACTOCENTRIC DISTANCE OF THE SUN AND ITS ROTATION VELOCITY (II)

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Abstract

The galactocentric distance of the sun is evaluated to be $R_0=8.3 \pm 0.9$ kpc by the comparison of the rotation velocity functions which are derived from the radial velocities of 113 cepheids and from that of the neutral hydrogen atoms, respectively. With the proper motions of 79 cepheids, this rotation velocity function yields the rotation velocity of the sun around the centre of the galaxy, $\omega_0=34.0 \pm 5.9$ km sec $^{-1}$ kpc^{-1}. The so-called unstable zone for cepheids between $R=8.3\sim 8.4$ kpc does not appear in the residual velocities.

1. Introduction.

This is the continuation of the work reported in the previous paper (Sinzi, 1960; hereafter designated as Paper I), in which the galactocentric distance of the sun, R_0, is derived from the kinematical data only on cepheids, whereas in the present method the data of the neutral hydrogen atoms are also employed.

In deriving the rotation velocity function $u(R)$ from the maximum radial velocities of 21cm emission, R is obtained by

$$R=R_0^* \sin\lambda, \tag{1}$$

where R_0^* is adopted value of the galactocentric distance of the sun, whereas in the case of stars, R, and hence $\varDelta R=R-R_0^*$, is computed from their r's and λ's. If we take correct value of R_0, the forms of the function $u(R)$ should be same for both cases. On the contrary, for any other value of R_0^*, resulting value of R should differ each other for 21cm emission and stars, and hence forms of $u(R)$ or $u(\varDelta R)$ should also differ each other.

2. $u(R)$ from 21cm emissions.

The data of $v_{\max}$ by Gum and Kerr (1958) are employed to derive the form of $u(R)$. At first, taking R_0 as unity, we determine the $u(R)$ curve under the condition that the curve passes on the position of the sun, i.e.

$$u=(1-|\sin\lambda|)(\alpha+\beta\,|\sin\lambda|), \tag{2}$$

where

$$u=v_{\max}/|\sin\lambda|, \tag{3}$$

and is shown in Fig. 1.

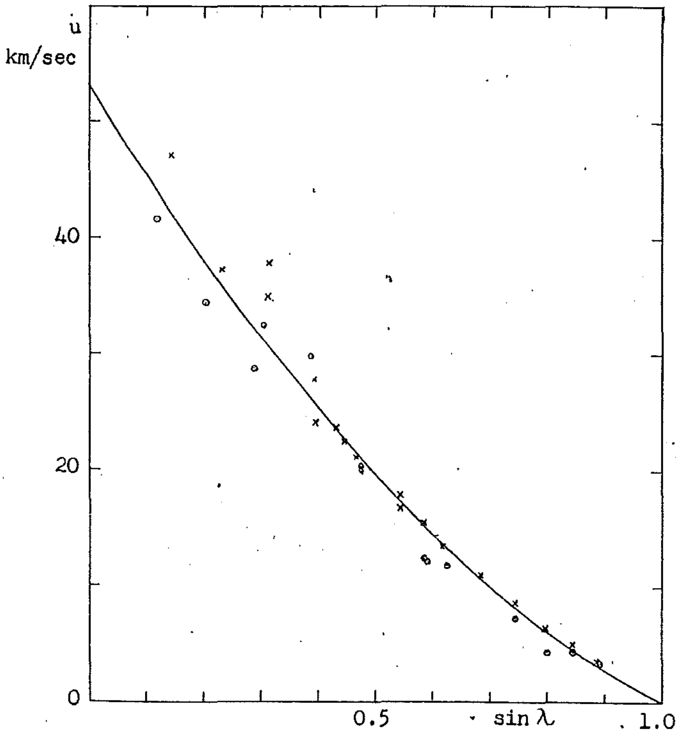

Fig. 1. Rotation Velocity Function by 21cm Observation.
$\odot$: $\lambda=296^\circ\sim360^\circ$, $\times$: $\lambda=0^\circ\sim62^\circ$

The least squares solutions are: (km sec^{-1})

$$\alpha=533\pm11, \quad \beta=-281\pm19 \text{ (p.e.)},$$

where probable errors are estimated from residuals for $|\sin\lambda|>0.4$, out of which, as recognised in Fig. 1, observed values of u scatter much.

Then, (2) becomes

$$\begin{array}{l} u=533-813\,|\sin\lambda|+281\sin^2\lambda. \\ \quad\ \pm11\ \ \pm22 \qquad\quad \pm19 \qquad \text{(p.e.)} \end{array} \tag{4}$$

$u(\Delta R)$ from cepheids.

In order to employ the cepheids data obtained by systematic observations as much as possible, we take 113 cepheids as shown in Table 1.

TABLE 1. NUMBERS AND WEIGHTS OF CEPHEIDS USED.

		RADIAL VELOCITIES.	
		Joy (1939)	Stibbs (1955)
Distances	Gascoign and Eggen (1957)	19 (0.5)	1 (0.75)
	Walraven, Muller and Oosterhoff (1958)	43 (0.75)	50 (1.0).

The distances of Gascoign and Eggen stars are recalculated so as to make them consistent with those of Walraven, Muller and Oosterhoff's stars by reducing the observed $(P\text{-}V)_E$ to SCI through the following relations (Walraven, Muller and Oosterhoff, 1958):

TABLE 2 a. BASIC DATA FOR JOY'S RADIAL VELOCITY STARS.

Js	Star	λ	$b^{\rm II}$	r	Dist.	$v_{\rm rp}$	u	ΔR_7	ΔR_8	ΔR_9	$v_{\rm p}$	z
		°	°	kpc		km/sec	km/sec	kpc	kpc	kpc	km/sec	pc
1	U Aql	31.0	−11.6	0.54	W	+4.8	+95.1	−0.45	−0.46	−0.46	−1.3	−109
2	SZ 〃	35.7	−2.4	1.79	〃	+22.8	+39.1	−1.35	−1.37	−1.38	−2.6	−75
3	TT 〃	36.0	−3.2	0.91	〃	+13.3	+22.7	−0.72	−0.72	−0.72	+1.5	−51
4	FF 〃	49.1	+6.3	0.33	〃	−0.1	−0.1	−0.22	−0.22	−0.22	−3.6	+36
5	FM 〃	44.3	+1.0	0.82	〃	+1.9	+2.7	−0.57	−0.56	−0.57	−8.8	+14
6	FN 〃	38.6	−3.1	1.11	〃	+21.3	+34.2	−0.84	−0.84	−0.85	+6.0	−60
7	v336 〃	34.2	−2.2	1.81	〃	+24.8	+44.2	−1.41	−1.42	−1.43	−0.7	−69
8	η 〃	41.1	−13.0	0.24	〃	−3.2	−5.0	−0.17	−0.17	−0.17	−5.2	−54
9	RT Aur	183.0	+8.9	0.51	E	+12.1			+0.51		+11.3	+79
10	RX 〃	165.8	−1.3	1.31	〃	−28.5			+1.27		−20.5	−30
11	RX Cam	146.0	+4.7	0.78	E	−37.4	−67.2	+0.66	+0.66	+0.66	−26.7	+64
12	RY CMa	226.0	+0.3	1.23	W	+23.8	−33.1	+0.90	+0.89	+0.89	+6.0	+6
13	TV 〃	227.2	−2.3	2.23	〃	+25.0	−34.1	+1.67	+1.65	+1.64	−4.3	−89
14	TW 〃	229.1	+0.1	2.46	〃	+52.8	−69.9	+1.81	+1.79	+1.78	+20.8	+5
15	RW Cas	129.1	−4.6	2.85	E	−62.4	−80.8	+2.07	+2.05	+2.02	−26.5	−228
16	SU 〃	133.1	+8.4	0.36	〃	−6.0	−8.3	+0.25	+0.25	+0.25	+0.4	+53
17	δ Cep	105.2	+0.5	0.26	〃	−9.0	−9.3	+0.08	+0.07	+0.08	−5.3	+2
18	X Cyg	77.0	−4.3	0.85	〃	+20.3	+20.9	−0.14	−0.15	−0.15	+17.9	−64
19	SU 〃	64.7	+2.6	0.87	W	−22.7	−25.1	−0.32	−0.33	−0.33	−29.9	+39
20	DT 〃	76.8	−10.8	0.44	E	+9.6	+10.0	−0.09	−0.09	−0.09	+8.8	−82
21	W Gem	197.4	+3.3	0.81	W	−12.6	+42.3	+0.78	+0.78	+0.78	−19.2	+47
22	ζ 〃	195.8	+11.9	0.24	E	−3.7		+0.23	+0.23	+0.23	−5.9	+49
23	BB Her	43.3	+6.8	II	W	+105.5	+154.9					
24	Y Lac	98.8	−4.0	2.37	E	−10.6	+11.8	+0.72	+0.68	+0.65	+31.1	−166
25	Z 〃	105.8	−1.6	1.71	〃	−18.8	−19.5	+0.65	+0.63	+0.61	−1.0	−48
26	RR 〃	105.7	−2.0	2.11	〃	−28.8	−29.9	+0.84	+0.81	+0.79	−6.8	−74
27	BG 〃	93.2	−9.2	1.78	〃	−11.7	−11.8	+0.32	+0.29	+0.27	−2.0	−285
28	T Mon	203.6	−2.6	0.81	W	+18.8	−47.0	+0.75	+0.75	+0.75	+10.2	−36
29	SV 〃	203.7	−3.7	2.39	〃	+13.2	−32.9	+2.24	+2.23	+2.23	−6.5	−155
30	TX 〃	214.1	−1.0	3.96	〃	+37.3	−66.5	+3.52	+3.50	+3.48	+2.8	−67
31	AC Mon	221.8	−1.9	2.41	W	+26.6	−39.9	+1.94	+1.93	+1.92	−3.2	−80
32	Y Oph	20.6	+10.1	0.41	〃	+7.5	+21.4	−0.37	−0.38	−0.37	+4.1	+72
33	RS Ori	196.6	+0.3	1.56	〃	+29.8			+1.50		+19.1	+8
34	SV Per	162.5	−1.5	2.21	E	−16.4	−54.5	+2.13	+2.13	+2.13	−2.0	−57
35	X Pup	236.1	−0.8	2.52	W	+48.0	−57.8	+1.67	+1.64	+1.62	+14.9	−35
36	RS 〃	252.4	−0.2	1.24	〃	+6.9	−7.2	+0.46	+0.45	+0.44	−6.5	−4
37	VW 〃	235.4	−0.6	3.91	〃	+10.5	−12.8	+2.77	+2.71	+2.67	−34.8	−39
38	VZ 〃	243.4	−3.3	4.38	〃	+35.7	−40.0	+2.78	+2.71	+2.64	−13.5	−254
39	WW 〃	237.4	+1.0	4.39	〃	+73.8	−87.6	+3.08	+3.01	+2.96	+24.9	+75
40	WY 〃	241.8	+2.7	4.54	〃	+31.3	−35.6	+2.99	+2.91	+2.84	−18.8	+213
41	WZ Pup	241.9	+3.3	3.72	W	+51.4	−58.4	+2.34	+2.29	+2.24	+7.4	+216
42	AD Pup	241.9	0.0	4.00	〃	+54.4	−61.7	+2.56	+2.49	+2.44	+8.1	0
43	S Sge	55.3	−6.1	0.58	〃	+2.4	+2.9	−0.31	−0.32	−0.32	−3.9	−61
44	Y Sgr	12.8	−2.1	0.47	〃	+8.4			−0.46		+5.7	−17
45	VY 〃	10.1	−1.0	2.33	〃	+5.4			−2.29		−8.8	−40
46	WZ 〃	12.1	−1.3	1.54	〃	+0.6			−1.50		−9.6	−34
47	XX 〃	15.0	−1.9	1.36	E	+14.4			−1.31		+3.7	−45
48	YZ 〃	17.7	−7.1	0.92	W	+30.0	+99.4	−0.88	−0.88	−0.88	+22.2	−114
49	AP 〃	8.1	−2.4	0.88	〃	−7.1			−0.87		−10.7	−36
50	AY 〃	13.2	−2.4	1.80	〃	−14.4			−0.57		−18.0	−74
51	BB Sgr	14.6	−9.0	0.70	W	+18.4			−0.68		+13.6	−109
52	v350 〃	13.6	−7.7	0.91	〃	+20.4			−0.88		+14.4	−122
53	X Sct	19.0	−1.6	1.78	〃	+19.3	+59.2	−1.64	−1.65	−1.65	+1.6	−50
54	Y 〃	24.0	−0.8	1.45	〃	+19.4	+47.7	−1.30	−1.31	−1.31	+2.6	−20
55	Z 〃	26.8	−0.7	2.27	〃	+42.6	+94.5	−1.92	−1.94	−1.95	−12.8	−27
56	RU 〃	28.2	+0.3	1.71	〃	−1.9	−4.0	−1.45	−1.46	−1.47	−24.1	+9
57	SS 〃	25.2	−1.8	0.95	〃	−1.2	−2.8	−0.84	−0.85	−0.85	−11.8	−29
58	UZ 〃	19.2	−1.5	2.63	〃	+24.4	+74.2	−2.39	−2.41	−2.42	−4.0	−68
59	ST Tau	193.1	−8.1	1.23	〃	−11.7			+1.20		−18.7	−173
60	T Vul	72.4	−10.2	0.58	E	+9.3	+9.9	−0.16	−0.16	−0.16	+6.6	−103
61	U Vul	56.1	−0.3	0.56	W	+1.7	+2.1	−0.29	−0.30	−0.30	−4.2	−3
62	X 〃	63.9	−1.3	0.99	E	−0.3	−0.3	−0.38	−0.38	−0.39	−9.0	−23
63	SV 〃	63.9	+0.3	1.48	〃	+10.4	+11.6	−0.51	−0.53	−0.54	−2.4	+7

TABLE 2 b. BASIC DATA FOR STIBBS' RADIAL VELOCITY STARS.

Ss	Star	λ	b^{II}	r	Dist.	v_{rp}	u	ΔR_7	ΔR_8	ΔR_9	v_{p}	z
		°	°	kpc		km/sec	km/sec	kpc	kpc	kpc	km/sec	pc
1	v496 Aql	28.4	−7.1	0.69	W	+17.5	+37.1	−0.60	−0.60	−0.60	+9.7	−86
2	*l* Car	283.0	−7.0	0.27	″	+6.3	−6.5	−0.05	−0.06	−0.06	+6.2	−33
3	U ″	289.1	+0.1	1.34	″	−5.3	+5.6	−0.32	−0.33	−0.35	+2.3	+3
4	V ″	274.9	−12.3	0.77	″	+6.3	−6.3	−0.03	−0.03	−0.04	+5.4	−164
5	Y ″	285.6	−0.3	1.48	″	−12.3	+12.8	−0.25	−0.27	−0.28	−6.5	−7
6	ER ″	290.1	+1.5	0.90	″	−23.4	+24.9	−0.25	−0.27	−0.27	−17.7	+23
7	GI ″	290.3	+2.5	1.51	″	−26.6	+28.4	−0.37	−0.38	−0.40	−17.6	+66
8	IT ″	291.5	−1.1	1.28	″	−19.3	+20.8	−0.37	−0.38	−0.39	−10.4	−24
9	UX ″	284.8	+0.2	1.56	″	+3.8	−3.9	−0.22	−0.25	−0.27	+9.2	+5
10	VY ″	286.5	+1.2	1.39	″	−73.2	+7.6	−0.26	−0.27	−0.29	−1.4	+29
11	V Cen	316.5	+3.3	0.74	W	−21.2	+30.9	−0.53	−0.52	−0.53	−11.6	+43
12	AZ ″	292.7	−0.2	1.46	″	−17.2	+18.6	−0.42	−0.44	−0.46	−6.6	−4
13	UZ ″	294.9	−1.0	1.65	″	−9.6	+10.6	−0.52	−0.54	−0.56	+3.6	−28
14	XX ″	309.6	+4.6	1.41	″	−16.1	+20.9	−0.80	−0.82	−0.83	+2.3	+113
15	v339 ″	313.4	−0.5	1.41	″	−24.4	+33.6	−0.88	−0.89	−0.91	−5.3	−13
16	v378 ″	306.1	−0.3	1.27	″	−19.4	+24.0	−0.67	−0.68	−0.69	−4.0	−6
17	v381 ″	310.9	+4.3	1.13	″	−31.0	+41.1	−0.68	−0.69	−0.69	−16.3	+85
18	v419 ″	292.1	+4.3	1.24	″	−21.7	+23.5	−0.37	−0.38	−0.40	−12.8	+93
19	R Cru	299.7	+1.1	0.87	″	−16.7	+19.2	−0.38	−0.39	−0.39	−8.1	+17
20	S ″	303.4	+4.5	0.70	″	−8.5	+10.2	−0.37	−0.36	−0.37	−1.0	+55
21	T Cru	299.4	+0.3	0.62	W	−9.3	+10.7	−0.28	−0.28	−0.28	−3.7	+3
22	X ″	302.3	+3.7	1.25	″	−27.3	+32.4	−0.58	−0.59	−0.60	−14.4	+81
23	AG ″	301.7	+3.1	1.35	″	−7.0	+8.2	−0.60	−0.62	−0.63	+7.7	+73
24	β Dor	270.8	−32.8	0.25	″	−3.2	+3.8	+0.01	+0.01	+0.01	−5.0	−136
25	R Mus	301.9	−6.6	0.81	″	+0.3	−0.4	−0.39	−0.40	−0.40	+8.9	−93
26	S ″	299.4	−7.6	0.63	″	+7.6	−8.8	−0.28	−0.29	−0.29	−13.5	−83
27	RT ″	296.4	−5.2	1.49	″	−6.4	+7.2	−0.52	−0.54	−0.56	+6.6	−136
28	S Nor	327.6	−5.4	0.66	″	+6.0	−11.2	−0.55	−0.56	−0.56	+14.2	−62
29	BF Oph	357.2	+8.6	0.79	″	−21.0			−0.79		−19.9	+119
30	κ Pav	327.7	−25.4	II	″	+38.2			II			
31	AP Pup	255.3	−5.7	0.98	W	+5.6	−5.8	+0.31	+0.31	+0.30	−4.4	−97
32	AT ″	254.3	−1.6	1.67	″	+16.8	−17.4	+0.62	+0.60	+0.59	−0.4	−47
33	WX ″	241.5	−1.3	2.34	″	+40.2	−45.7	+1.38	+1.35	+1.33	+9.9	−54
34	U Sgr	13.7	−4.4	0.57	″	+15.3			−0.55		+11.8	−44
35	W ″	1.5	−4.0	0.40	″	−16.7			−0.40		−17.4	−28
36	X ″	1.2	+0.2	0.32	″	−3.5			−0.32		−3.7	+1
37	RV Sco	350.5	+5.7	0.76 E	″	−14.9			−0.75		−11.4	+75
38	RY ″	356.5	−3.4	0.93	E	−9.5			−0.93		−7.8	−55
39	v482 ″	354.3	+0.1	0.96	W	+20.1			−0.96		+23.0	+2
40	v500 ″	359.0	−1.3	1.23	″	−4.2			−1.23		−3.5	−28
41	v636 Sco	343.1	−5.2	0.64	W	+7.0	−24.2	−0.60	−0.61	−0.61	+11.9	−58
42	R TrA	316.8	−7.7	0.66	″	−9.2	+13.5	−0.47	−0.47	−0.47	−0.8	−88
43	S TrA	321.9	−8.2	0.79	″	+7.7	−12.6	−0.60	−0.60	−0.61	+17.8	−113
44	U TrA	323.1	−8.1	1.25	″	−12.3	+20.7	−0.95	−0.96	−0.96	+4.8	−176
45	T Vel	265.4	−3.8	1.08	″	−2.7	+2.7	+0.18	+0.16	+0.15	−9.0	−71
46	V ″	276.5	−3.3	1.07	″	−38.3	+38.6	−0.04	−0.05	−0.06	−38.7	−62
47	AH ″	262.2	−6.2	0.49	″	+14.6	−14.8	+0.09	+0.09	+0.09	+10.2	−53
48	AX ″	263.1	−6.9	1.10	″	+12.8	−13.0	+0.21	+0.20	+0.19	+5.5	−132
49	BG ″	271.8	−1.7	0.71	″	+1.8	−1.8	+0.01	+0.01	+0.01	−0.3	−21
50	RZ ″	262.8	−1.1	1.51	″	+14.6	−14.7	+0.35	+0.32	+0.31	+4.0	−29
51	SV ″	286.0	+3.2	2.03	W	−1.6	+1.7	−0.27	−0.31	−0.34	+5.5	+114
52	SX ″	265.4	−1.4	1.80	″	+20.4	−20.5	+0.36	+0.33	+0.31	+9.5	−43

λ: Difference in the galactic longitudes between the direction of the galactic center and the star.
r: Distance to the star.
Dist: Photometry employed for the derivation of the distance;
W: Walraven et al., E: Gascoign and Eggen.
v_{rp}: Peculiar radial velocity.
ΔR_7, ΔR_8, ΔR_9: Differences in the galactocentric distances between the star and the sun, assuming $R_0^*=7$, 8 and 9 kpc, respectively.
v_{p}: Residual velocity.
z: Distance of the star from the galactic plane.

$$1.216\,(\log I_B+\log I_Y)+0.88\,(\log I_B-\log I_Y)=11.537-P_E,$$
$$1.227\,(\log I_B+\log I_Y)-1.25\,(\log I_B-\log I_Y)=10.510-V_E,$$
$$\log I_B=4.6790-0.4046\,SP_g+0.0056\,SP_v,$$
$$\log I_Y=4.2551-0.0157\,SP_g-0.3838\,SP_v,$$

and

$$SCI=SP_g-SP_v,$$

from which photographic space absorption A_{pg} is computed:

$$A_{\text{pg}}=\chi\,SCI_{\text{Excess}}=3.5\,(SCI_{\max}-SCI_{\text{obs}}).$$

The equatorial and galactic coordinates are taken from the revised edition of the General Catalogue of Variable Stars (Kukarkin, Parenago et al., 1958), reducing latitudes to the new system by means of the relation:

$$\Delta b=b^{II}-b^{I}=1.^\circ 50\cos(12.^\circ 431+l^{I}).$$

Radial velocities are corrected for Dyer's (1956) basic solar motion $v_\odot=15.3$ km sec^{-1} to $A=262^\circ.4$ and $D=+20^\circ.3$.

In Table 2 basic data are shown where $\Delta R=R-R_0^*$ are computed for $R_0^*=7$, 8 and 9 kpc, respectively.

Putting

$$u=\alpha+\beta\Delta R+\gamma(\Delta R)^2, \tag{5}$$

the method of least squares are carried out for 92 stars which satisfy the condition $|\sin\lambda\cos b|>0.3$. The coefficients of the normal equations and their solutions are given in Table 3.

TABLE 3. COEFFICIENTS OF NORMAL EQUATIONS AND THEIR SOLUTIONS FOR $u=\alpha+\beta\cdot\Delta R+\gamma(\Delta R)^2$.

	$[aa]$	$[ab]$	$[ac][bb]$	$[bc]$	$[cc]$	$[am]$	$[bm]$	$[cm]$	$[mm]$			
R_0^*	ΣW	ΔR	ΔR^2	ΔR^3	ΔR^4	u	$u\cdot\Delta R$	$u\cdot\Delta R^2$	u^2	α	β	γ
kpc 7	76	+3.40	92.57	+134.87	516.45	+98.3	−2210.9	−2278.2	82962	−1.5 ±1.6	−28.7 ± 1.7	+3.4 ±0.8
8	76	+2.35	90.84	+125.34	488.15	+98.3	−2195.3	−2148.0	82962	−1.7 ±1.6	−28.6 ± 1.7	+3.3 ±0.8
9	76	+1.53	89.51	+118.15	465.85	+98.3	−2183.3	−2049.0	82962	−1.9 ±1.6	−28.6 ± 1.7	+3.2 ±0.8

4. Determination of R_0.

Putting

$$\xi=1-\sin\lambda=\Delta R/R_0^*,$$

both (4) and (5) can be written

$$y_1=\alpha_1+\beta_1\xi+\gamma_1\xi^2 \qquad \text{for 21cm emission,}$$
$$y=\alpha+\beta\xi+\gamma\xi^2 \qquad \text{for cepheids.}$$

Then, the difference of these two kinds of empirical relations yields

$$\begin{aligned} Q &= \int_0^{-\xi} (y-y_1)^2 d\xi \\ &= -(\alpha-\alpha_1)^2\xi + (\alpha-\alpha_1)(\beta-\beta_1)\xi^2 - \frac{1}{3}\{2(\alpha-\alpha_1)(\gamma-\gamma_1) \\ &\quad + (\beta-\beta_1)^2\}\xi^3 + \frac{1}{2}(\beta-\beta_1)(\gamma-\gamma_1)\xi^4 - \frac{1}{5}(\gamma-\gamma_1)^2\xi^5. \end{aligned}$$

The integrals are performed between the limits $\varDelta R=0\sim 2.4$ kpc in which both 21cm and cepheids observations were made. The results are:

R_0	7	8	9 kpc
Q/R_0	0.98	0.03	0.33 km² sec⁻².

Assuming Q/R_0 to be a quadratic function of R_0, we obtain

$$R_0=8.3 \text{ kpc},$$

at which Q/R_0 takes minimum value. From Table 3 we may assume: (km sec^{-1})

$$\begin{array}{ccccc} u= & -1.8 & -28.6\,\varDelta R & +3.3\,(\varDelta R)^2. & \\ \pm 12 & \pm 1.6 & \pm 1.7 & \pm 0.8 & \text{(p.e.)} \end{array} \tag{6}$$

At $\sin\lambda=1$ and $\varDelta R=0$, the increments of u for R are, from (4) and (6), -29.8 and -28.6 km sec^{-1} kpc^{-1}, respectively, giving the probable errors of R_0 at $\varDelta R=0$ to be ± 0.42 and ± 0.82 kpc, respectively. We take the error of the R_0 determination as the square root of the sum of these errors. Then, we have

$$R_0=8.3\pm 0.9 \text{ kpc},$$

which agrees well with the current value.

5. Determination of ω_0.

The method introduced in Paper I, Section 4 is applied for all 79 cepheids whose proper motions are provided by Morgan and Weaver (1956). Among these stars, 53 stars are included in Walraven, Muller and Oosterhoff's observation. The distances of other 15 stars are reduced from Gascoign and Eggen's data to the Dutch system through the method described in Section 3. The distances of the remaining 9 stars are shown in Table 1 of Janak's (1958a) paper with the absorption A_{pg} which perhaps due to the photographic colour observation by Badalyan (1956). These distances are recalculated by the following process. There are 96 population I cepheids which are common to Walraven, Muller and Oosterhoff's– and Janak's papers. The empirical relation between SCI_{Excess} and $A_{\text{pg(Janak)}}$ are obtained by the method of least squares, i. e.

$$\begin{array}{ccc} 3.5\,SCI_{\text{Excess}}= & 0.68 & +1.00\,A_{\text{pg (Janak)}}, \\ \pm .95 & \pm .07 & \pm .05 \quad \text{(p.e.)} \end{array}$$

by which the distances consistent with those by Walraven, Muller and Oosterhoff are obtained.

In Table 4, the basic data are shown. Employing the rotation velocity function $u(\varDelta R)$ of (6) and assigning the weights through the formula

$$W=\frac{1}{\sigma_\mu^2+\left(\frac{\sigma_t}{\kappa r}\right)^2}$$

with $\sigma_t=10\,\text{km sec}^{-1}$, least squares solutions by (11) of Paper I,

$$\omega(\varDelta R)\cos b\sin\varphi=\kappa\varDelta n\cos\alpha-\kappa\mu_\delta$$
$$+\frac{R_0\varDelta\omega(\varDelta R)}{r}(\cos\lambda\sin\varphi-\sin\lambda\sin b\cos\varphi)$$

yields

$$\omega_0=34.0\pm5.9\,\text{km sec}^{-1}\,\text{kpc}^{-1},$$

for 72 stars which satisfy the condition $|\sin\varphi\cos b|>0.25$.

Following the definitions of $A=-\frac{1}{2}R_0\omega_0'$ and $B=A-\omega_0$, the results obtained in this section yield the values of the dynamical parameters:

TABLE 4. BASIC DATA FOR MORGAN-WEAVER'S PROPER MOTION STARS.

WM	Star	μ_δ	$\mu_{\text{p.e.}}$	r_{WM}	r_{GE}	r_{WMO}	r_{I}	r_{J}	r_{adopt}	u_{c}	$\varDelta R_8$	ω
		$\times10^{-3}$ ″	$\times10^{-3}$ ″	kpc	kpc	kpc	kpc	kpc	kpc	km/sec	kpc	km/sec kpc
1	TU Cas	0	6	0.85				1.10	0.73	−12.2	+0.38	
2	SU Cas	2	2	0.59	0.40			0.36	0.36	− 8.8	+0.25	−130
3	RW Cam	−20	11	1.60				2.10	2.19	−43.7	+1.87	−195
4	RX Cam	−10	9	1.34	0.86			0.85	0.78	−19.2	+0.66	−105
5	SZ Tau	− 6	4	0.49	0.53	23.4		0.43	0.47	−14.5	+0.47	− 42
6	AW Per	− 2	6	1.19				0.85	0.93	−25.1	+0.91	− 16
7	RX Aur	− 7	6	1.38	1.36			1.21	1.31	−32.8	+1.27	− 43
8	ST Tau	− 6	5	1.31	1.45	1.24	1.28 :	—	1.24	−31.3	+1.20	− 15
9	SV Mon	− 5	10	3.09		2.39		1.41	2.39	−49.2	+2.23	− 3
10	RS Ori	− 9	8	1.59		1.56	1.86	1.45	1.56	−37.3	+1.50	− 20
11	T Mon	− 4	5	1.92	0.77	0.86	1.17	0.88	0.86	−21.5	+0.75	+ 8
12	RT Aur	−13	2	0.47	0.55			0.34	0.52	−15.5	+0.51	− 32
13	W Gem	− 8	5	1.11		0.81		0.60	0.81	−22.1	+0.78	− 7
14	ζ Gem	− 3	1	0.44	0.26			0.24	0.25	− 8.2	+0.23	+ 37
15	RY CMa	+ 5	14	1.39		1.23	1.53	1.10	1.23	−24.7	+0.82	+ 66
16	VX Pup	−17	13	1.30			1.62 :	0.69	0.72	−12.9	+0.41	− 53
17	X Pup	+17	12	2.87		2.51	3.62	2.56	2.51	−39.8	+1.64	+134
18	AP Pup	+17	9	1.27		1.67	1.24	0.93	1.67	−10.4	+0.31	+139
19	AH Vel	+ 4	4	0.67	0.53	0.49	0.58 :	0.40	0.49	− 4.4	+0.09	+ 70
20	RS Pup	+ 2	14	2.47	1.01	1.24	1.31	1.60	1.24	−14.0	+0.45	+ 61
21	V Car	− 4	6	1.55		0.79	1.20 :	1.46	0.79	− 0.9	−0.03	+ 30
22	RZ Vel	− 6	6	1.70	1.57	1.51	—	0.90	1.51	−10.7	+0.32	+ 22
23	SW Vel	0	9	2.75	2.46	2.58	3.20 :		2.58	−17.0	+0.57	+ 61
24	SX Vel	−11	6	2.14		1.80	2.08	1.32	1.80	−10.8	+0.33	− 8
25	V Vel	0	10	1.37	1.12	1.07	1.33 :	1.26	1.07	− 0.4	−1.50	+ 81
26	*l* Car	+ 6	3	0.87	0.24	0.27	0.36	0.65	0.27	− 0.1	−0.06	+140
27	UX Car	− 9	14	1.49		1.56	1.88	1.00	1.56	+ 5.6	−0.25	+ 47
28	Y Car	+ 5	12	1.29		1.48	1.59 :	0.95	1.48	+ 1.1	−0.27	+188
29	U Car	+ 7	5	1.65		1.34	1.07	1.12	1.34	+ 8.0	−0.33	+264
30	ER Car	− 6	6	1.26		0.90	1.18	0.80	0.90	+ 6.1	−0.27	+123

TABLE 4. BASIC DATA FOR MORGAN-WEAVER'S PROPER MOTION STARS. (CONTINUED)

WM	Star	μ_δ	$\mu_{p.e.}$	r_{WM}	r_{GE}	r_{WMO}	r_I	r_J	r_{adopt}	u_c	ΔR_8	ω
		$\times 10^{-3}$ ″	$\times 10^{-3}$ ″	kpc	kpc	kpc	kpc	kpc	kpc	km/sec	kpc	km/sec kpc
31	S Mus	−15	6	1.17	0.70	0.64	0.76	0.80	0.64	+ 6.8	−0.29	+ 23
32	T Cru	+ 3	6	1.20	0.67	0.62	0.82 :	0.84	0.62	+ 6.5	−0.28	
33	R Cru	− 7	6	1.18	1.02	0.87	0.97 :	0.64	0.87	+ 9.9	−0.39	
34	R Mus	− 6	6	1.17	0.92	0.82	0.96 :	0.96	0.82	+10.1	−0.40	
35	S Cru	−11	6	1.03		0.70	0.85	0.49	0.70	+ 8.9	−0.36	
36	X X Cen	− 4	7	1.73		1.41	1.82	1.57	1.41	+23.9	−0.82	
37	v381 Cen	+ 5	7	1.27		1.13	1.30	1.03	1.13	+19.5	−0.69	
38	V Cen	− 9	6	1.07		0.74	0.87	0.83	0.74	+14.0	−0.52	− 23
39	R TrA	−15	5	0.90		0.67	0.73	0.67	0.67	+12.3	−0.47	+ 54
40	S TrA	− 2	6	0.99		0.80	0.89	0.75	0.80	+16.6	−0.60	− 29
41	U TrA	0	9	1.16		1.27	1.30 :	0.84	1.27	+28.7	−0.96	− 37
42	S Nor	+ 3	5	1.03		0.66	0.81	0.79	0.66	+15.3	−0.56	− 49
43	R V Sco	− 2	7	0.86		0.76	0.88	0.64	0.76	+21.5	−0.75	+ 17
44	B F Oph	+ 8	5	0.95		0.80	0.88	0.74	0.80	+22.8	−0.79	− 43
45	X Sgr	− 9	1	0.45	0.34	0.32	0.37	0.33	0.32	+ 7.7	−0.32	+ 68
46	R Y Sco	+20	9	1.97	0.95		1.32	3.26	0.93	+27.6	−0.93	− 86
47	Y Oph	− 8	5	1.25	0.44	0.41	0.46 :	0.86	0.41	+ 9.9	−0.38	+ 60
48	W Sgr	− 1	2	0.49	0.40	0.40	0.43	0.37	0.40	+25.0	−0.40	+ 69
49	A P Sgr	− 4	4	1.12		0.88	0.96	0.49	0.88	+25.6	−0.87	+ 53
50	W Z Sgr	− 9	6	3.81	1.51	1.54	2.04	2.00	1.54	+48.5	−1.50	+ 83
51	Y Sgr	−13	3	0.62	0.54	0.47	0.54	0.42	0.47	+12.1	−0.46	+101
52	X X Sgr	− 6	9	1.98	1.51			1.59	1.36	+41.4	−1.31	+ 68
53	U Sgr	−11	3	0.83	0.63	0.57	0.68	0.68	0.57	+14.9	−0.55	+ 94
54	S S Sct	+ 4	7	1.05	1.09	0.95	1.12	0.98	0.95	+24.9	−0.85	+ 16
55	v350 Sgr	−10	13	1.10	0.95	0.92	0.98	0.80	0.92	+25.9	−0.88	+ 94
56	Y Z Sgr	0	4	1.50	0.94	0.93	1.30 :	0.97	0.93	+25.9	−0.88	+ 43
57	B B Sgr	−20	6	1.24		0.71	0.84 :	0.80	0.71	+19.1	−0.68	+149
58	F F Aql	− 8	3	0.44	0.37	0.33	—	0.31	0.33	+ 4.7	−0.22	+ 71
59	S Z Aql	− 5	7	3.81	1.81	1.79		2.39	1.79	+43.6	−1.37	+ 67
60	T T Aql	−13	3	1.70	0.92	0.91	1.07 :	1.58	0.91	+20.1	−0.72	+109
61	U Aql	− 2	4	0.87	0.65	0.55	0.50 :	0.50	0.55	+12.1	−0.46	+ 61
62	U Vul	−10	4	1.08		0.56		0.83	0.56	+ 7.1	−0.30	+ 94
63	S V Cyg	+10	9	0.96	1.00	0.87		0.70	0.87	+ 8.0	−0.33	− 15
64	S V Vul	+ 1	4	1.75	1.33			1.53	1.48	+14.3	−0.53	+ 36
65	η Aql	− 7	1	0.36	0.27	0.24	0.28 :	0.20	0.24	+ 3.2	−0.17	+ 88
66	S Sge	− 4	2	0.73	0.61	0.58	0.74 :	0.55	0.58	+ 7.7	−0.32	+ 71
67	X Vul	+10	7	1.50	1.13			1.10	0.99	+ 9.1	−0.38	− 11
68	C D Cyg	+17	10	3.91				2.38	2.56	+11.1	−0.43	− 53
69	S Z Cyg	+ 7	8	3.43				2.65	2.78	− 7.4	+0.20	+ 14
70	X Cyg	+ 2	4	1.67	0.84			1.00	0.85	+ 2.5	−0.15	+ 50
71	T Vul	+ 3	3	0.62	0.62			0.46	0.59	+ 2.9	−0.16	+ 48
72	V X Cyg	+ 5	14	3.31				3.40	3.42	− 9.3	+0.27	+ 34
73	D T Cyg	+ 3	3	0.38	0.49			0.35	0.45	+ 0.8	−0.09	+ 51
74	V Z Cyg	+15	14	1.65				1.38	1.46	− 6.6	+0.17	− 18
75	Y Lac	+ 2	13	2.53	2.56			2.20	2.38	−19.7	+0.68	+ 98
76	δ Cep	+ 3	1	0.31	0.29			0.21	0.26	− 3.8	+0.07	+110
77	R R Lac	−20	11	2.77	2.26			1.87	2.11	−22.8	+0.81	+362
78	V Lac	+14	12	1.98				1.41	1.53	−16.8	+0.56	+ 7
79	X Lac	−12	14	1.76				1.05	1.10	−12.2	+0.38	+280

r_{WM}: Distance adopted by Morgan and Weaver (1956).
r_{GE}: Distance adopted by Gascoign and Eggen (1957).
r_{WMO}: Distance adopted by Walraven et al. (1958).
r_I : Distance adopted by Irwin (unpublished).
r_J : Distance adopted by Janak (1958a).
u_c : Computed rotation velocity function.
ω: Angular rotation velocity.

$$A= 14.3\pm0.9\,\mathrm{km\ sec^{-1}\ kpc^{-1}},$$
$$B=-19.7\pm5.9\,\mathrm{km\ sec^{-1}\ kpc^{-1}}$$

and

$$V_0=279 \quad \mathrm{km\ sec^{-1}\ for}\ R_0=8.2\ \mathrm{kpc},$$
$$V_0=282\pm56 \quad \mathrm{km\ sec^{-1}\ for}\ R_0=8.3\pm0.9\ \mathrm{kpc}.$$

Weaver (1955a, b, 1959) points out the existence of inconsistency in the currently adopted values of R_0, ω_0, and V_0, remarking the imperative need to investigate the value of R_0. The above value of $R_0\omega_0$ suggests that this inconsistency may be attributed to the underestimations of ω_0 as well as of R_0.

6. The "unstable zone".

Janak(1957), using the Lindblad's diagram, points out that cepheids lying in the region $R=8.1\sim8.7$ kpc show the galactic orbit of large eccentricities. Later, in his kinematical analysis, he (1958b) refined this "unstable zone" to $R=8.30\sim8.40$ kpc. In the 12th column of Table 3, the residual velocities are showns which are obtained by subtracting the terms of the galactic rotation from the peculiar radial velocities. This quantity is plotted against R in Fig. 2. It is difficult to distinguish this untable zone from the figure.

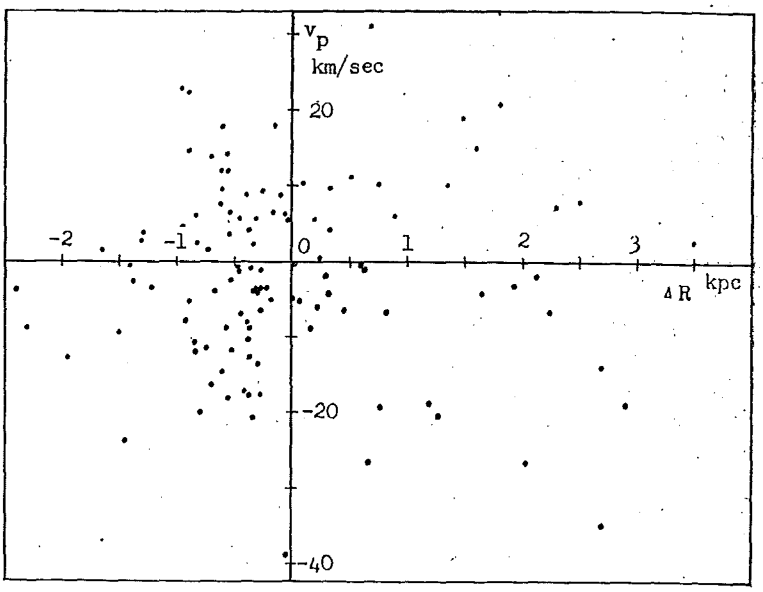

Fig. 2. Residual Velosity Distribution against the Galactocentric Distance.

7. Conclusion.

Since the values of R appeared in the expression of $u(R)=v_{\max}/\sin\lambda$ by 21cm emission include one parameter λ, while for $u(\Delta R)=v/\sin\lambda\cos b$ include two parameters r and λ, R_0 can be fixed by adjusting R_0 so as to yields same shapes of u for both cases.

Using the 21cm data by Gum and Kerr, radial velocities of cepheids by Joy and by Stibbs, and data on distances by Walraven, Muller and Oosterhoff, and by Gascoign and Eggen, we obtain:

$$R_0=8.3\pm0.9 \text{ kpc},$$

further, adding the distance by Janak the proper motions data by Morgan gives:

$$\omega_0=34.0\pm5.9 \text{ km sec}^{-1} \text{ kpc}^{-1},$$

whence

$$V_0=282\pm56 \text{ km sec}^{-1}.$$

These values are favourable to exclude the inconsistency in the currently adopted values of R_0, ω_0, and V_0.

The so-called unstable zone for cepheids at $R=8.3\sim8.4$ kpc does not appear in the residual velocity.

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